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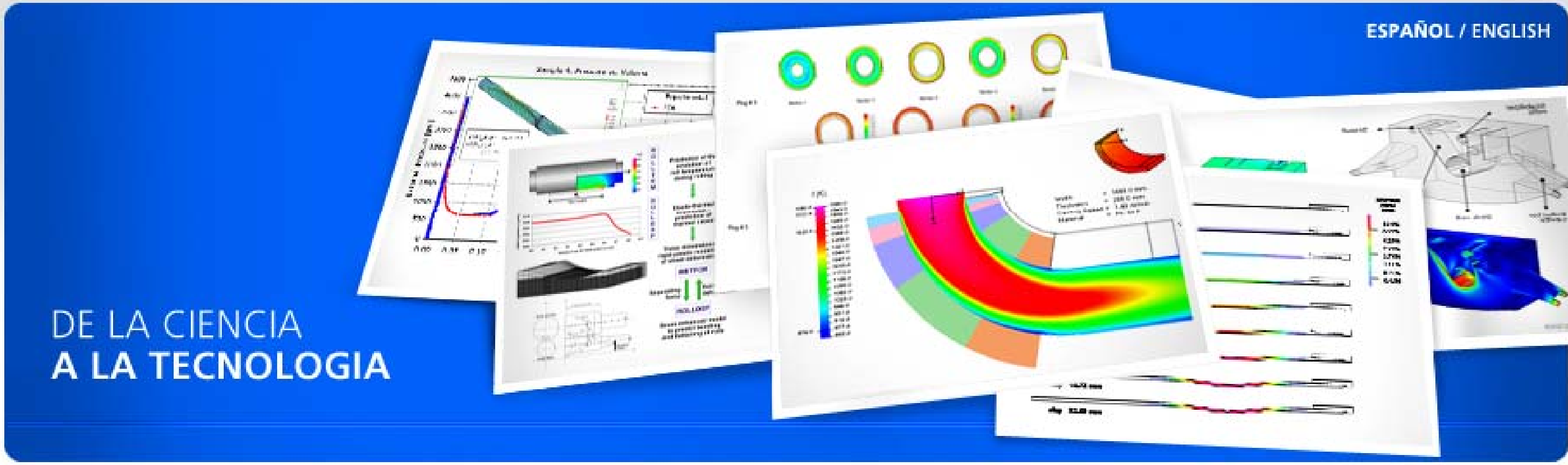

FEM in Heat Transfer

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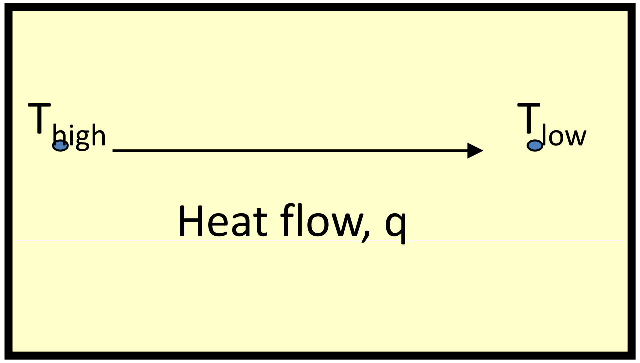
Introduction: Heat transfer

There are 3 mechanism of heat transfer: Conduction, Convection and Radiation.

Conduction is defined as transfer of heat occurring through intervening matter without bulk motion of the matter. The increased motion of a particle with an energy level (temperature) higher energizes adjacent molecules which are at lower energy levels.



Fourier law (1682)

$$\underline{q} = -k \nabla T$$

$$q_x = -k\left(\frac{\partial T}{\partial x}\right) \quad ; \quad q_y = -k\left(\frac{\partial T}{\partial y}\right)$$

$$q_z = -k\left(\frac{\partial T}{\partial z}\right)$$

Introduction: Heat transfer

Conduction

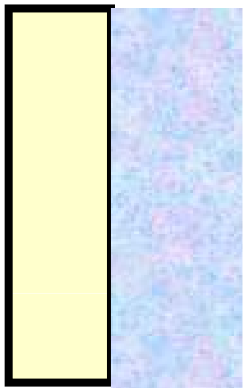
k: is the thermal conductivity

Metals	Ag	Cu	Al	Fe	Steel
k [W/m-K]	420	390	200	70	50

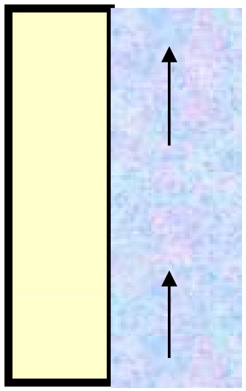
Non-metals	H_20	Air	Engine oil	H_2	Brick	Wood	Cork
k [W/m-K]	0.6	0.026	0.15	0.18	0.4 -0 .5	0.2	0.04

Introduction: Heat transfer

Convection heat transfer is due to a flowing fluid. It occurs when a liquid or gas (fluids) comes in contact with a material of a different temperature.
Natural convection occurs when the flow of a liquid or gas is primarily due to density differences within the fluid due to heating or cooling of that fluid.
Forced convection occurs when the flow of fluid (liquid or gas) is primarily due to pressure differences.



Natural convection



Forced convection

Newton law of cooling

$$q_n = h\,\Delta T$$

$$q_n = h\left(T_{surface} - T_{medium}\right)$$

h is the convective heat transfer coefficient, [W/m^{2}K]

Introduction: Heat transfer

Convection

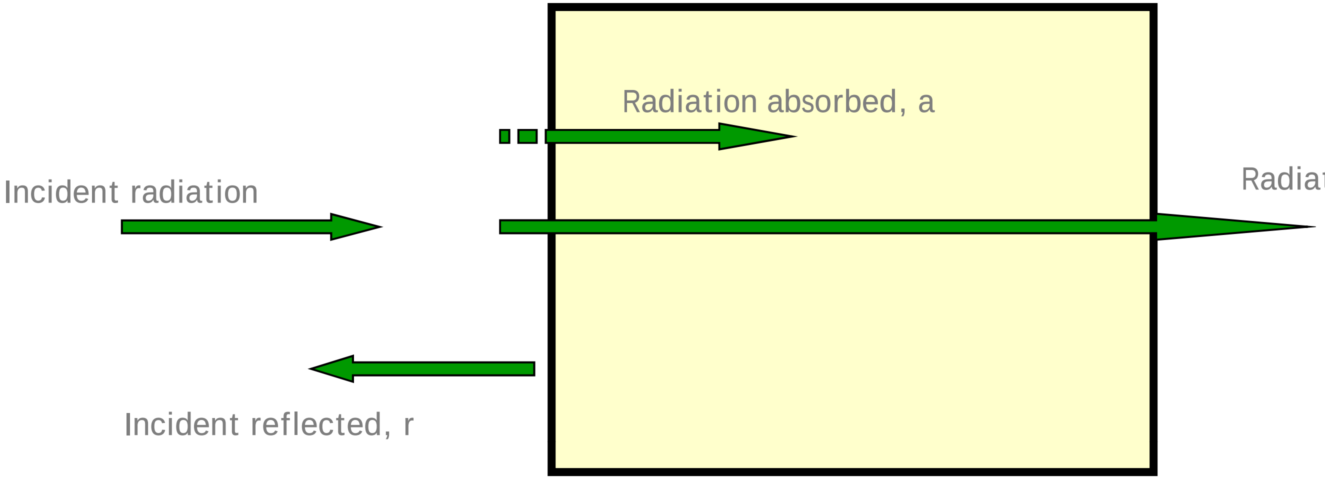
Situation	$\overline{h}$, W/m^2K
Natural convection in gases	
• 0.3 m vertical wall in air, $\Delta T = 30°C$	4.33
Natural convection in liquids	
• 40 mm O.D. horizontal pipe in water, $\Delta T = 30°C$	570
• 0.25 mm diameter wire in methanol, $\Delta T = 50°C$	4,000
Forced convection of gases	
• Air at 30 m/s over a 1 m flat plate, $\Delta T = 70°C$	80
Forced convection of liquids	
• Water at 2 m/s over a 60 mm plate, $\Delta T = 15°C$	590
• Aniline-alcohol mixture at 3 m/s in a 25 mm I.D. tube, $\Delta T = 80°C$	2,600
• Liquid sodium at 5 m/s in a 13 mm I.D. tube at 370°C	75,000
Boiling water	
• During film boiling at 1 atm	300
• In a tea kettle	4,000
• At a peak pool-boiling heat flux, 1 atm	40,000
• At a peak flow-boiling heat flux, 1 atm	100,000
• At approximate maximum convective-boiling heat flux, under optimal conditions	10^6
Condensation	
• In a typical horizontal cold-water-tube steam condenser	15,000
• Same, but condensing benzene	1,700
• Dropwise condensation of water at 1 atm	160,000

Introduction: Heat transfer

Radiation heat transfer is the transmission of energy through space without the necessary presence of matter.

Radiation is the transfer of heat from one object to another by means of electro-magnetic waves. Radiative heat transfer does not require that objects be in contact or that a fluid flow between those objects.

Radiative heat transfer occurs in the void of space (that's how the sun warms us).



$$\alpha + \rho + \tau = 1$$

Introduction: Heat transfer

Radiation

$$q_n = \sigma \varepsilon \Delta T^4 \quad ; \quad q_n = \sigma \varepsilon \left(T_{surface} - T_{medium}\right)^4$$

An ideal thermal radiator is called a "black body“, *a = 1*

Real bodies radiate less effectively than black bodies.

ε is the emittance, $\varepsilon = \dfrac{\text{radiation from real body at T}}{\text{radiation from black body at T}}$

σ is the Stefan Boltzman constant, $\sigma = 5.67\,10^{-8}\,\dfrac{W}{m^2 K}$

Heat transfer equation

$$\underbrace{\rho\, C_p \frac{\partial T}{\partial t}}_{\text{Transient term}} + \underbrace{\rho\, C_p\, \underline{v} \cdot \underline{\nabla} T}_{\text{Convective term}} = \underbrace{\underline{\nabla} \cdot \left(\underline{\underline{k}} \cdot \underline{\nabla} T \right)}_{\text{Diffusive term}} + \underbrace{q_V}_{\text{Volumetric heat [W/m}^3\text{]}}$$

ρ density [Kg/m³]

C_p Specific heat [J/Kg °K]

$\underline{\underline{k}}$ Thermal conductivity [W/M°K]

$\underline{v}$ Velocity [m/s]

$$\boxed{\underline{q} = -k\, \underline{\nabla} T}$$

$\underline{q}$: heat flux [W/m2]

Heat transfer equation

Heat transfer equation in cartesian coordinates

$$\rho Cp \frac{\partial T}{\partial t} + \rho Cp \left(v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) =$$

$$\frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z} \right) + q_v$$

Heat transfer equation

Heat transfer equation in cylindrical coordinates

$$\rho Cp \frac{\partial T}{\partial t} + \rho Cp \left(v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) =$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(k\, r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left(k \frac{\partial T}{\partial \theta} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + q_v$$

Non-dimensional numbers

The **Biot number (Bi)** is used in transient heat transfer calculations. It is named by the French Jean-Baptiste Biot (1774-1862), and gives a simple index of the ratio of the heat transfer resistances *inside of* and *at the surface of* a body

$$Bi = \frac{h\,Lc}{k}$$

The **Prandtl number** (Pr) is the ratio of momentum diffusivity and thermal diffusivity. It is named by the German Ludwig Prandtl (1875-1953).

$$\mathrm{Pr} = \frac{\mu / \rho}{k / \rho Cp} = \frac{\mu\, Cp}{k}$$

The **Nusselt number** (Nu) is the ratio of convective to conductive heat transfer across (normal to) the boundary. It is named by the German Wilhelm Nusselt (1882-1957)

$$Nu = \frac{h\,L}{k_f}$$

Non-dimensional numbers

The **Stanton number (St)** is the ratio of heat transferred into a fluid to the thermal capacity of fluid. It is used to characterize heat transfer in forced convection flows.

$$St = \frac{Nu}{\mathrm{Re}\ \mathrm{Pr}}$$

The **Grashof number (Gr)** approximates the ratio of the buoyancy to viscous force acting on a fluid. It is named by the german Granz Grashof (1826-1893)

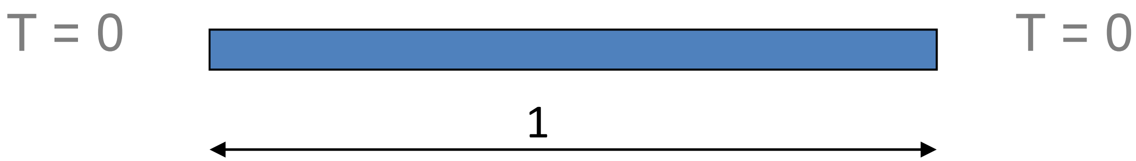
$$Gr = \frac{\rho\, \beta \left(T_w - T_\infty\right) L^3}{\left(\mu / \rho\right)^2}$$

The **Rayleigh number (Ra)** for a fluid is associated with buoyancy driven flow (or natural convection). When the Rayleigh number is below the critical value for that fluid, heat transfer is primarily in the form of conduction; when it exceeds the critical value, heat transfer is primarily in the form of convection. It is named by Lord Rayleigh (1842-1919)

$$Ra = Gr\ \mathrm{Pr}$$

The finite element method in heat transfer

Physical problem



Solid bar – 1D problem – Volumetric heat constant $q_v = cte$

Mathematical problem

$$\mathrm{I}\quad \begin{cases} k\,\dfrac{\partial^2 T}{\partial x^2} + q_v = 0 & 0 \le x \le 1 \\ T(0) = 0 \\ T(1) = 0 \end{cases}$$

The finite element method in heat transfer

The **strong form** of the contour values problem is :

$$Let \quad q_v : [0,1] \rightarrow \Re \quad finds \quad T : [0,1] \rightarrow \Re \quad like \quad is \quad fulfilled \qquad \mathrm{I}$$

Analytical Solution

$$T = -\frac{q_v}{k}\frac{x^2}{2} + C_1 x + C_2$$

$$T = \frac{q_v}{2k}\left[x - x^2\right]$$

Numerical model

$$k\frac{\partial^2 \tilde{T}}{\partial x^2} + q_v = R\left(\tilde{T}\right) \neq 0 \qquad 0 \leq x \leq 1$$

$$T \rightarrow \tilde{T}$$

residual

We need $R\left(\tilde{T}\right) \rightarrow 0$

The finite element method in heat transfer

Method of weighted residual

$$\int_0^1 \omega_i(x)\, R\left(\tilde{T}\right) dx = 0$$

The unknown function T is aproximate by

$$\omega_i(x) = \delta(x - x_i) \quad \rightarrow$$ Point collocation method

$$\omega_i(x) = h_i(x) \quad \rightarrow$$ Galerkin method

$$\omega_i(x) = R(x_i) \quad \rightarrow$$ Square minimum method

The finite element method in heat transfer

Bubnov- Galerkin method

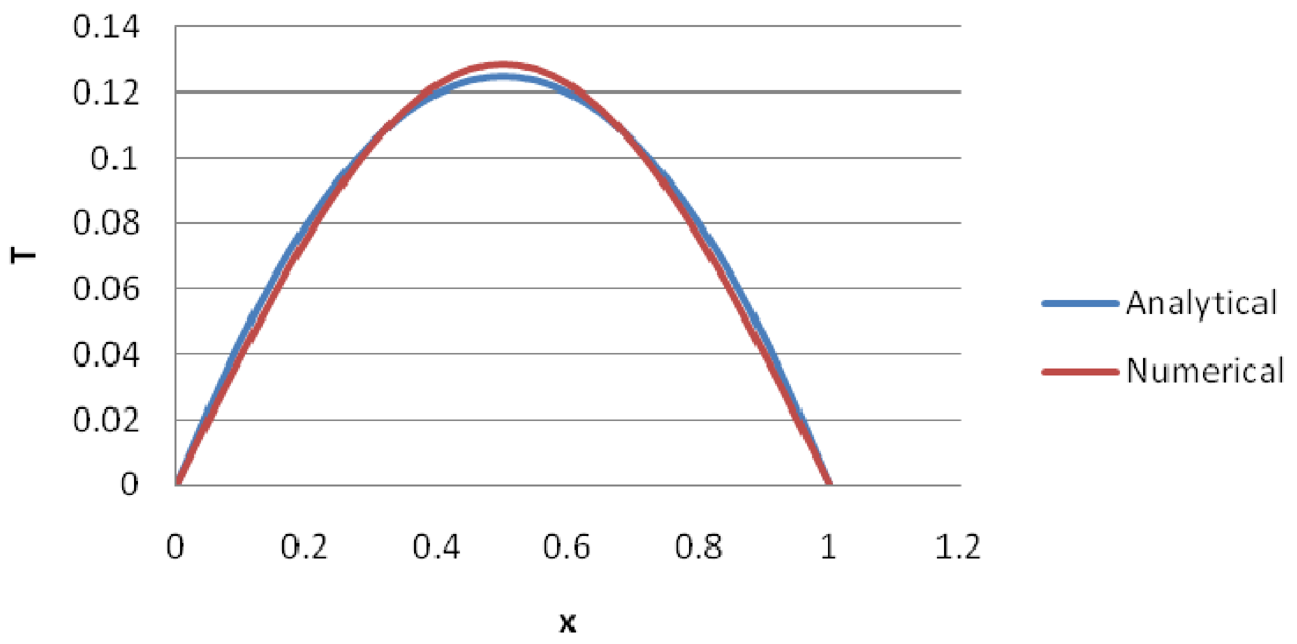
$$\tilde{T} = \underbrace{sen\left(\pi \; x\right)}_{h(x)}\hat{T} \qquad \frac{\partial \tilde{T}}{\partial x} = \pi \cos\left(\pi \; x\right)\hat{T} \qquad \frac{\partial^2 \tilde{T}}{\partial x^2} = -\pi^2 \sin\left(\pi \; x\right)\hat{T}$$

The unknow (number)

$$\int_0^1 \sin\left(\pi \; x\right) \left[-\pi^2 k \sin\left(\pi \; x\right)\hat{T} + q_v\right] dx = 0$$

$$\tilde{T} = \frac{4 q_v}{k \pi^3} \sin\left(\pi \; x\right)$$

The finite element method in heat transfer

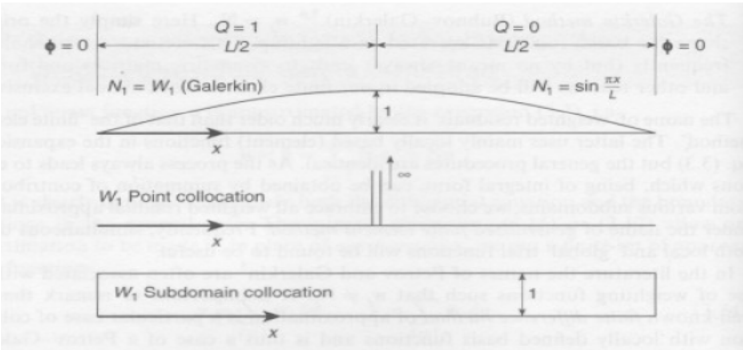


k=1

q_v=1

The finite element method in heat transfer

Bubnov- Galerkin method



Approximating with 1 function

From Zienkiewicz & Taylor, The Finite Element Method

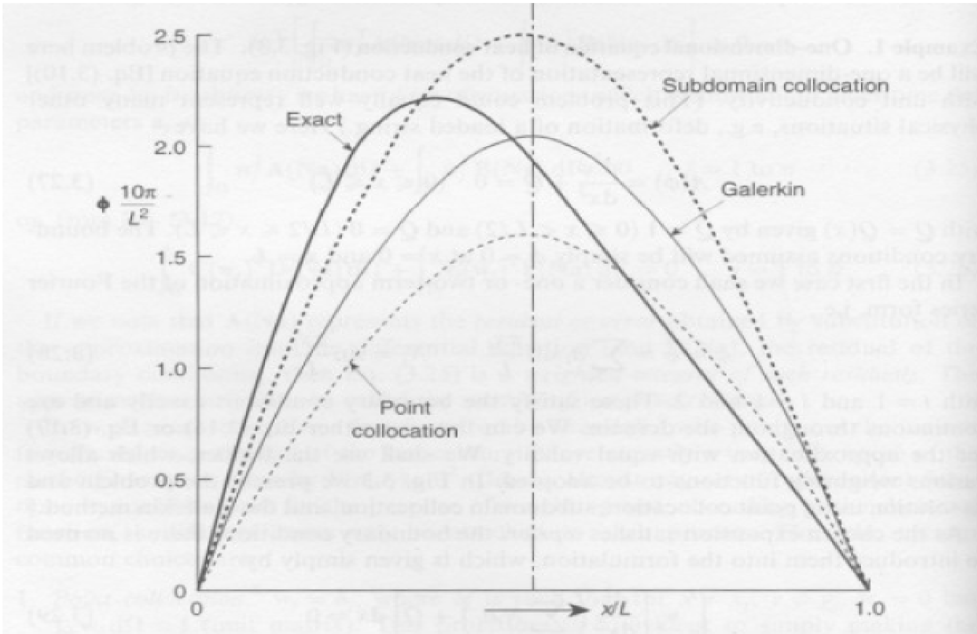


Fig. 3.3 One-dimensional heat conduction. (a) One-term solution using different weighting procedures.

The finite element method in heat transfer

Bubnov- Galerkin method

Notes for the approximate solution:

1. The "rigid" (essential) boundary conditions are strictly imposed.
2. The differential equation is not necessarily fulfilled at every point.
3. Improve the solution by increasing the number of interpolation functions "h_i"

The finite element method in heat transfer

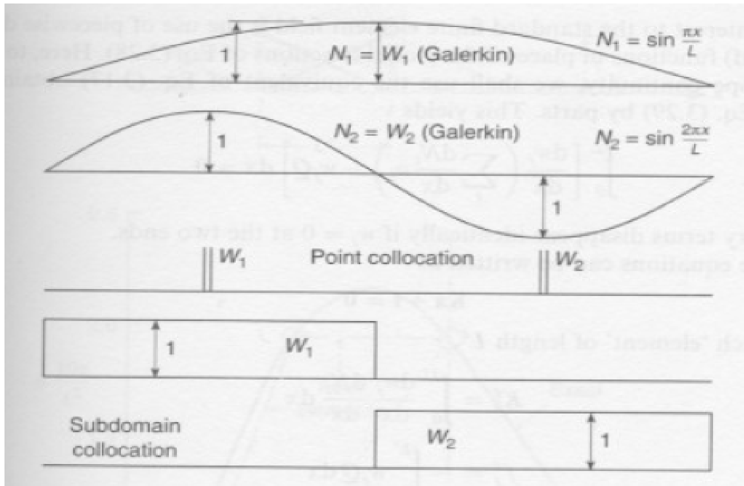
Bubnov- Galerkin method

Increase the number of interpolation functions

$\sin(\pi x/2)$	No (cannot represent the b.c.)
$\sin(\pi x)$	o.k.
$\sin(3\pi x/2)$	No (cannot represent the b.c.)
$\sin(2\pi x)$	o.k.

The finite element method in heat transfer

Bubnov- Galerkin method



Approximating with 2 functions

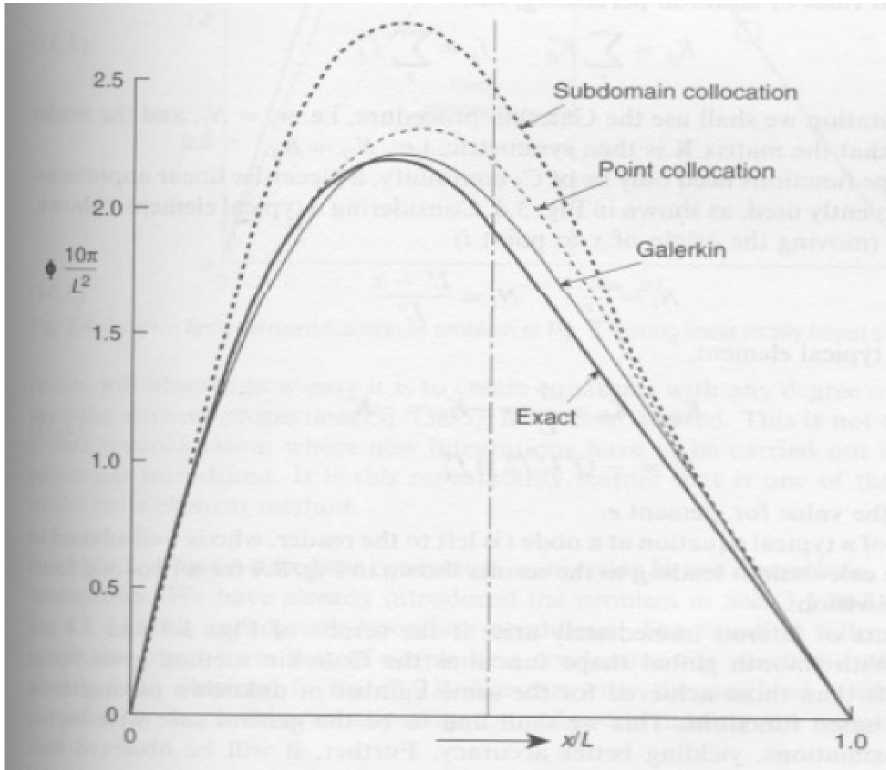


Fig. 3.3 (cont.) (b) Two-term solutions using different weighting procedures.

From Zienkiewicz & Taylor, The Finite Element Method

The finite element method in heat transfer

Bubnov- Galerkin method

$$k\frac{\partial^2 T}{\partial x^2} + q_v = 0 \qquad 0 \le x \le 1$$

$$T(0) = 0$$

$$T(1) = 0$$

$$\tilde{T} = \underline{H} \cdot \underline{\hat{T}} = h_1\hat{T}_1 + h_2\hat{T}_2 + ... + h_r\hat{T}_r$$

$$\int_0^1 \left(h_i \frac{\partial}{\partial x}\left(k\frac{\partial \tilde{T}}{\partial x}\right) + h_i\, q_v \right) dx = 0$$

Integration by parts

$$\int_0^1 \left(-\frac{\partial h_i}{\partial x}\left(k\frac{\partial \tilde{T}}{\partial x}\right) + h_i\, q_v \right) dx + h_i\, k\frac{\partial \tilde{T}}{\partial x}\Bigg|_0^1 = 0$$

$$\boxed{h_i(0) = h_i(1) = 0}$$

The finite element method in heat transfer

Bubnov- Galerkin method

$$\int_0^1 \left(\frac{\partial h_i}{\partial x} \sum_{j=1} \left(\frac{\partial h_j}{\partial x} \hat{T}_j \right) - h_i \, q_v \right) dx = 0 \quad \Rightarrow \quad \underline{\underline{K}} \bullet \underline{\hat{T}} = \underline{f}$$

$$K_{ij}^{(e)} = \int_0^{L^{(e)}} \frac{\partial h_i}{\partial x} \frac{\partial h_j}{\partial x} dx \qquad f_i^{(e)} = \int_0^{L^{(e)}} h_i \, q_v \, dx$$

$$K_{ij} = \sum_{(e)} K_{ij}^{(e)} \qquad f_i = \sum_{(e)} f_i^{(e)}$$

The finite element method in heat transfer

Bubnov- Galerkin method

$$k\frac{\partial^2 T}{\partial x^2} + q_v = 0 \qquad 0 \le x \le 1$$

$$T(0) = 0$$

$$-k\left.\frac{\partial T}{\partial x}\right|_1 = q_n$$

$$h_i(0)$$

$$\int_0^1 \left(-\frac{\partial h_i}{\partial x}\left(k\frac{\partial \tilde{T}}{\partial x}\right) + h_i\, q_v \right) dx + h_i\, k \left.\frac{\partial \tilde{T}}{\partial x}\right|_0^1 = 0$$

$$-k\left.\frac{\partial \tilde{T}}{\partial x}\right|_1 = q_n$$

$$\int_0^1 \left(-\frac{\partial h_i}{\partial x}\left(k\frac{\partial \tilde{T}}{\partial x}\right) + h_i\, q_v \right) dx - h_i\big|_1\, q_n = 0 \quad \Rightarrow \quad \underline{\underline{K}} \bullet \underline{\hat{T}} = \underline{F}$$

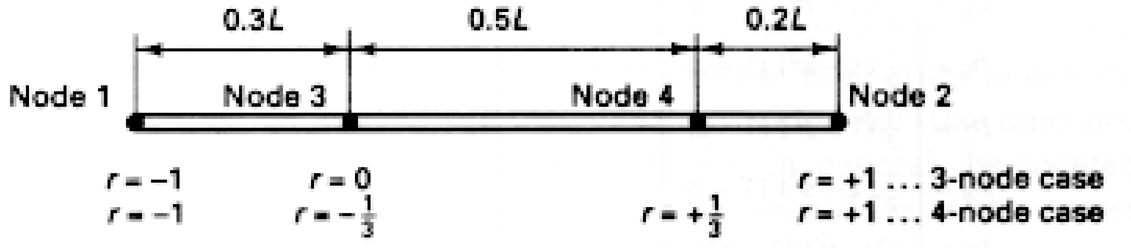
The finite element method in heat transfer

Bubnov- Galerkin method

$$K_{ij}^{(e)} = \int_0^{L^{(e)}} \frac{\partial h_i}{\partial x} \frac{\partial h_j}{\partial x} dx \qquad f_i^{(e)} = \int_0^{L^{(e)}} h_i\, q_v\, dx + h_i\big|_1\, q_n$$

$$K_{ij} = \sum_{(e)} K_{ij}^{(e)} \qquad f_i = \sum_{(e)} f_i^{(e)}$$

The finite element method in heat transfer



(a) 2 to 4 variable-number-nodes truss element

	Include only if node 3 is present	Include only if nodes 3 and 4 are present
$h_1 = \frac{1}{2}(1 - r)$	$-\frac{1}{2}(1 - r^2)$	$+\frac{1}{16}(-9r^3 + r^2 + 9r - 1)$
$h_2 = \frac{1}{2}(1 + r)$	$-\frac{1}{2}(1 - r^2)$	$+\frac{1}{16}(9r^3 + r^2 - 9r - 1)$
$h_3 = (1 - r^2)$		$+\frac{1}{16}(27r^3 + 7r^2 - 27r - 7)$
$h_4 = \frac{1}{16}(-27r^3 - 9r^2 + 27r + 9)$		

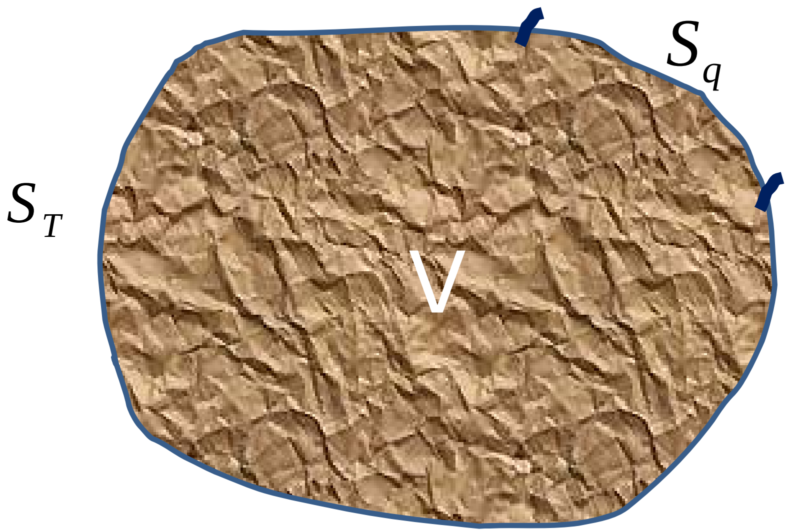
(b) Interpolation functions

Figure 5.3 Interpolation functions of two to four variable-number-nodes one-dimensional element

From Bathe, Finite Element Procedures

The finite element method in heat transfer

2D / 3D Problems



$$\rho Cp \frac{\partial T}{\partial t} + \rho Cp \left(v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) =$$

$$\frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z} \right) + q_v$$

Boundary conditions

$$\boxed{\begin{aligned} S_T \cap S_q &= 0 \\ S_T \cup S_q &= S \end{aligned}}$$

$$T = \breve{T} \quad on \quad S_T$$

$$k \underline{\nabla} T = -\underline{q} \quad on \quad S_q$$

S_q : heat flux prescribed

S_T : temperature prescribed

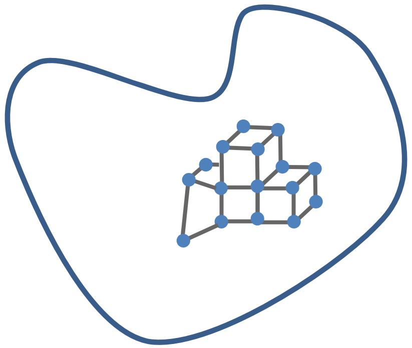
Initial conditions

$$T(x, y, z)\big|_{t=0}$$

The finite element method in heat transfer

2D/3D Problems

Elements and nodes



The interpolation functions inside an element

$$\tilde{T} = \sum_{1}^{NNOD} h_k \, T_k$$

T_k : *temperature at node "k"*

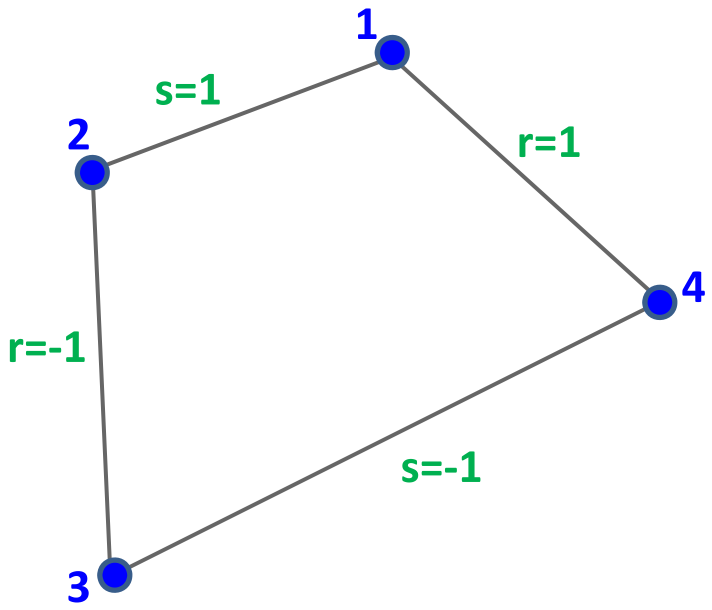
h_k : *interpolation function*

$h_k = 1$ *at node "k"*

$h_k = 0$ *at node* $\neq$ *"k"*

The finite element method in heat transfer

2D example



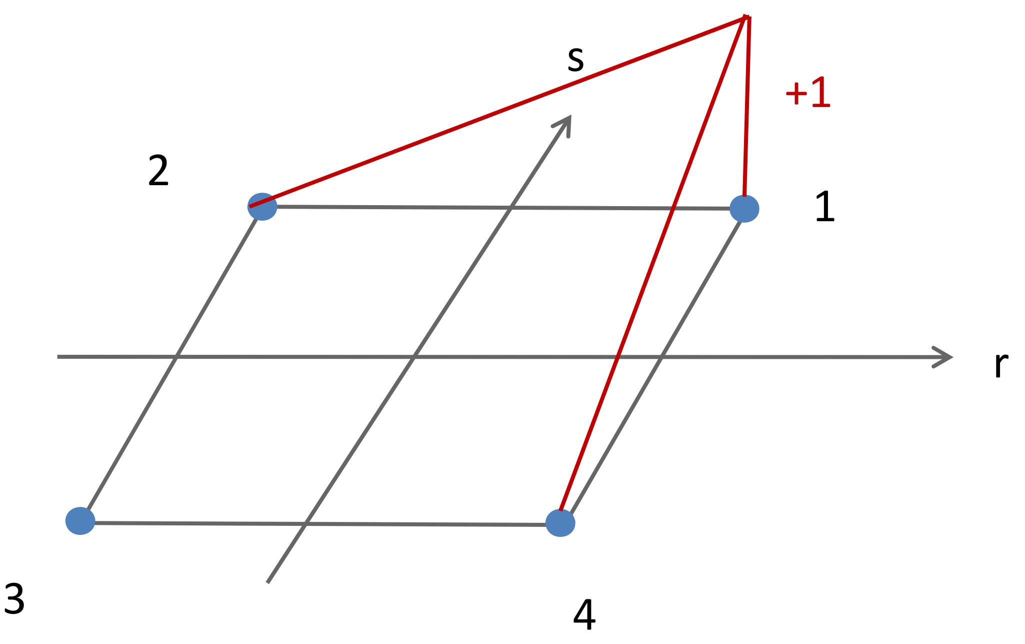
Natural coordinate system inside each element (r, s)

$$
\begin{aligned}
-1 &\leq r \leq 1 \\
-1 &\leq s \leq 1
\end{aligned}
$$

$$
\begin{aligned}
h_1(r,s) &= \frac{1}{4}(1+r)(1+s) \\
h_2(r,s) &= \frac{1}{4}(1-r)(1+s) \\
h_3(r,s) &= \frac{1}{4}(1-r)(1-s) \\
h_4(r,s) &= \frac{1}{4}(1+r)(1-s)
\end{aligned}
$$

The finite element method in heat transfer

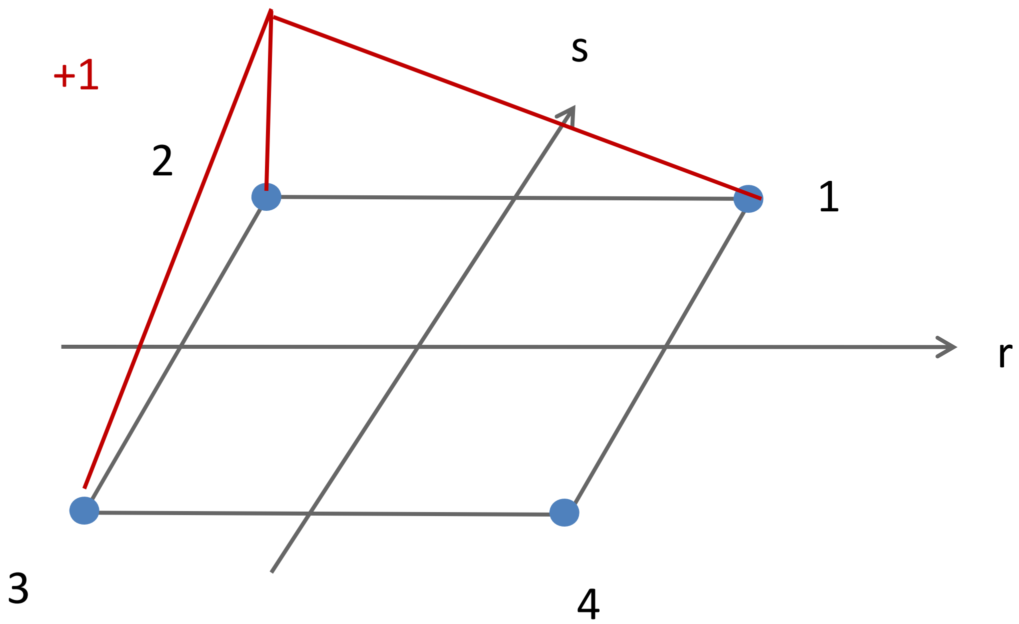
2D four-nodes element



$$\boxed{h1}$$

The finite element method in heat transfer

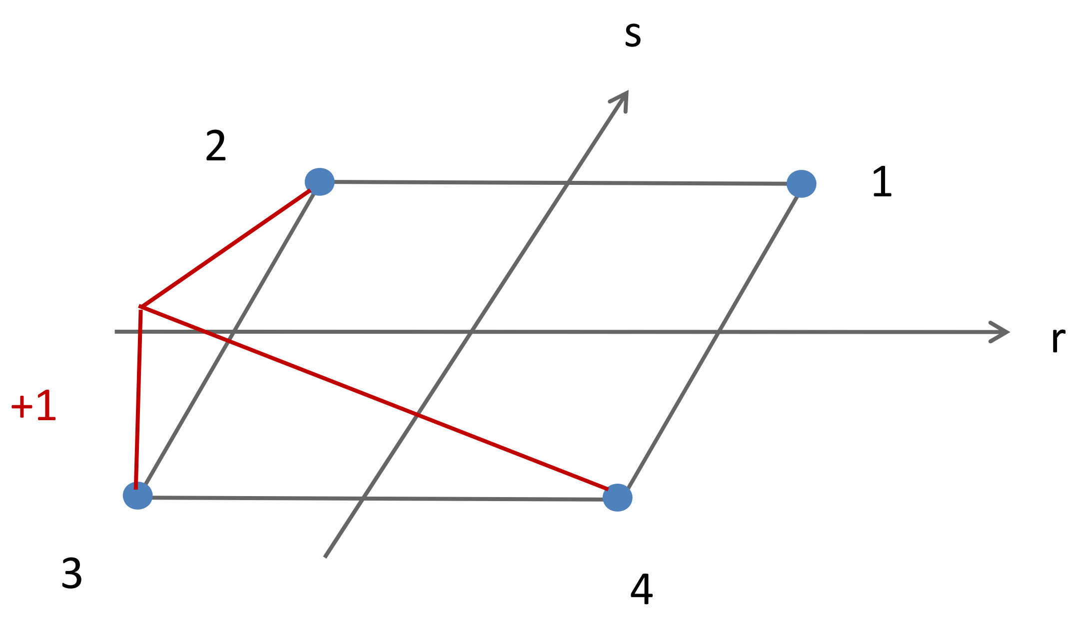
2D four-nodes element



$\boxed{h2}$

The finite element method in heat transfer

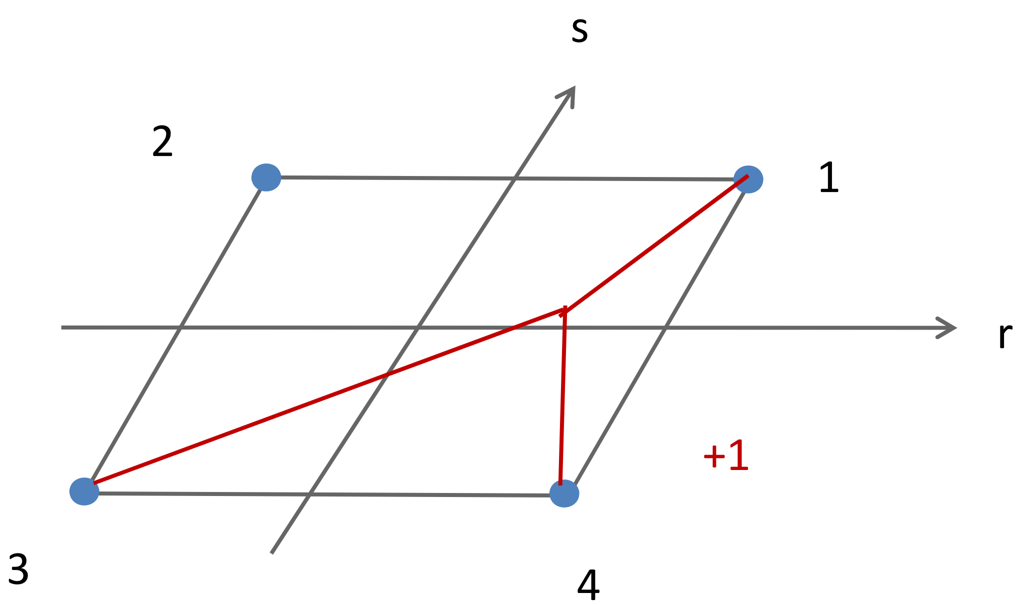
2D four-nodes element



$\boxed{h3}$

The finite element method in heat transfer

2D four-nodes element



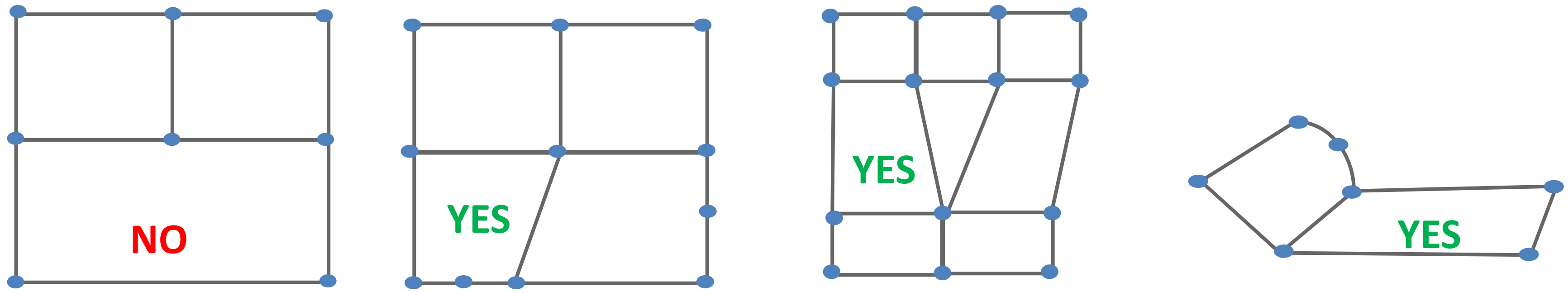
$\boxed{h4}$

The finite element method in heat transfer

$$\sum_{k=1}^{k=NNOD} h_k = 1$$

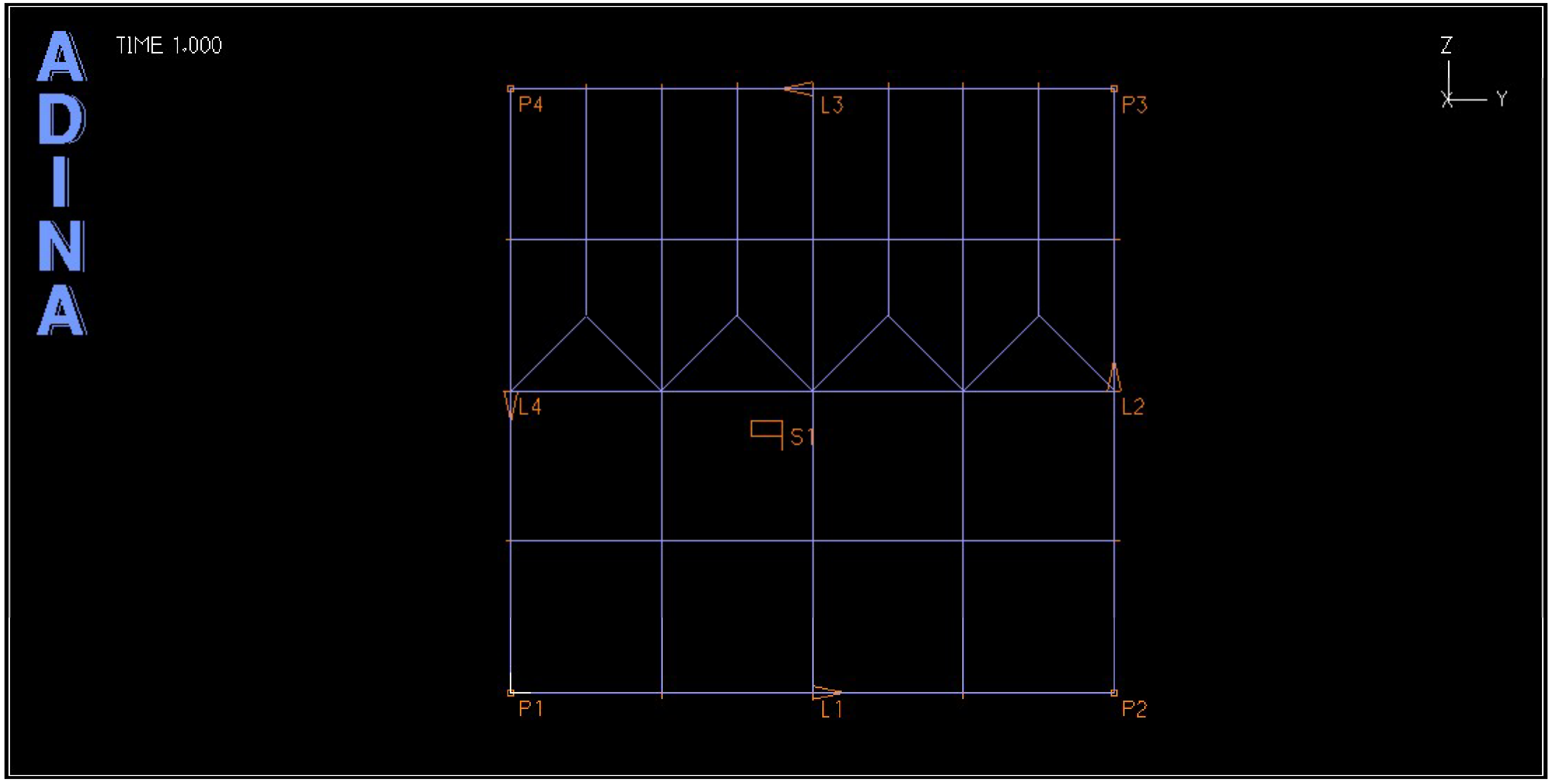
to be able to represent constant temperature situations

Are these meshes acceptable?



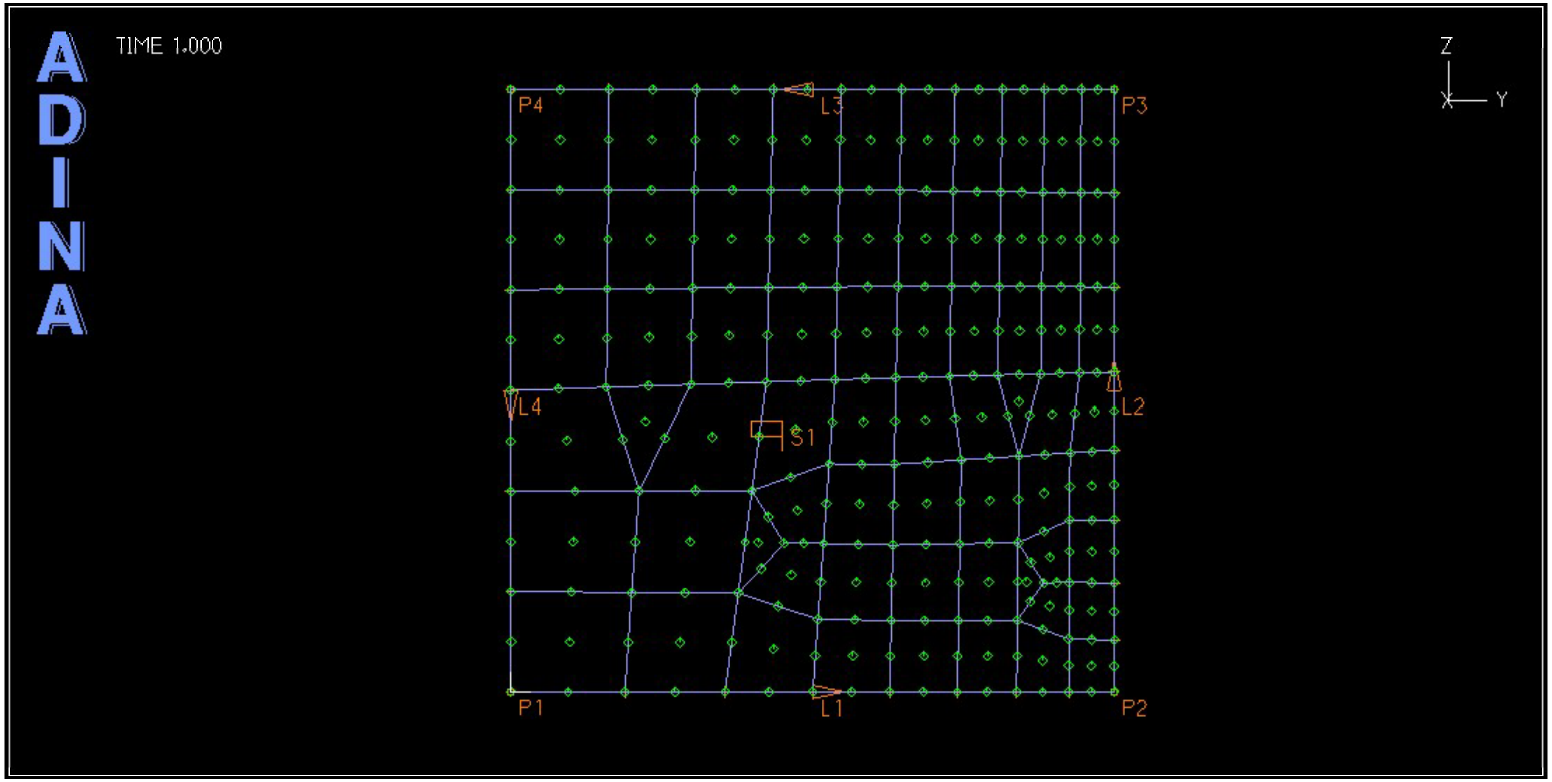
The finite element method in heat transfer

Good mesh



The finite element method in heat transfer

Good mesh



The finite element method in heat transfer

Good mesh

However!!!

Minimize the element distortions to have good predictive capability

Target for each element: $\det(\underline{J})=\text{const}$

The finite element method in heat transfer

Isoparametric elements

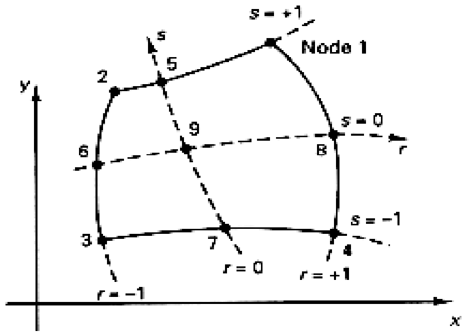
$$\tilde{T}(r,s,t) = h_k(r,s,t)\, T_k$$

$$x(r,s,t) = h_k(r,s,t)\, x_k$$

$$y(r,s,t) = h_k(r,s,t)\, y_k$$

$$z(r,s,t) = h_k(r,s,t)\, z_k$$

The finite element method in heat transfer



(a) 4 to 9 variable-number-nodes two-dimensional element

Include only if node i is defined

		$i=5$	$i=6$	$i=7$	$i=8$	$i=9$
$h_1 =$	$\frac{1}{4}(1+r)(1+s)$	$-\frac{1}{2}h_5$			$-\frac{1}{2}h_8$	$-\frac{1}{4}h_9$
$h_2 =$	$\frac{1}{4}(1-r)(1+s)$	$-\frac{1}{2}h_5$	$-\frac{1}{2}h_6$			$-\frac{1}{4}h_9$
$h_3 =$	$\frac{1}{4}(1-r)(1-s)$		$-\frac{1}{2}h_6$	$-\frac{1}{2}h_7$		$-\frac{1}{4}h_9$
$h_4 =$	$\frac{1}{4}(1+r)(1-s)$			$-\frac{1}{2}h_7$	$-\frac{1}{2}h_8$	$-\frac{1}{4}h_9$
$h_5 =$	$\frac{1}{2}(1-r^2)(1+s)$					$-\frac{1}{2}h_9$
$h_6 =$	$\frac{1}{2}(1-s^2)(1-r)$					$-\frac{1}{2}h_9$
$h_7 =$	$\frac{1}{2}(1-r^2)(1-s)$					$-\frac{1}{2}h_9$
$h_8 =$	$\frac{1}{2}(1-s^2)(1+r)$					$-\frac{1}{2}h_9$
$h_9 =$	$(1-r^2)(1-s^2)$					

(b) Interpolation functions

Figure 5.4 Interpolation functions of four to nine variable-number-nodes two-dimensional element

From Bathe, Finite Element Procedures

The finite element method in heat transfer

$h_1 = g_1 - (g_9 + g_{12} + g_{17})/2$

$h_2 = g_2 - (g_9 + g_{10} + g_{18})/2$

$h_3 = g_3 - (g_{10} + g_{11} + g_{19})/2$

$h_4 = g_4 - (g_{11} + g_{12} + g_{20})/2$

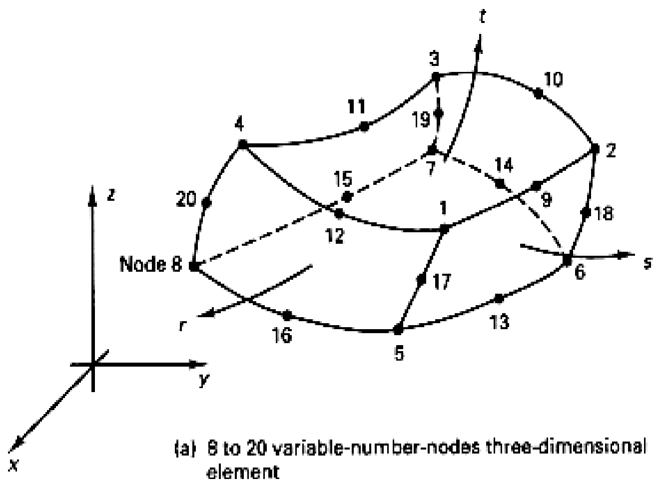
$h_5 = g_5 - (g_{13} + g_{16} + g_{17})/2$

$h_6 = g_6 - (g_{13} + g_{14} + g_{18})/2$

$h_7 = g_7 - (g_{14} + g_{15} + g_{19})/2$

$h_8 = g_8 - (g_{15} + g_{16} + g_{20})/2$

$h_j = g_j$ for $j = 9, \ldots, 20$



(a) 8 to 20 variable-number-nodes three-dimensional element

Figure 5.5 Interpolation functions of eight to twenty variable-number-nodes three-dimensional element

$g_i = 0$ if node i is not included; otherwise,

$g_i = G(r, r_i)\, G(s, s_i)\, G(t, t_i)$

$$\left.\begin{aligned} G(\beta, \beta_i) &= \tfrac{1}{2}(1 + \beta_i \beta) \quad \text{for } \beta_i = \pm 1 \\ G(\beta, \beta_i) &= (1 - \beta^2) \qquad \text{for } \beta_i = 0 \end{aligned}\right\} ; \beta = r, s, t$$

(b) Interpolation functions

Figure 5.5 *(continued)*

From Bathe, Finite Element Procedures

The finite element method in heat transfer

$$\left(\underline{\underline{\mathbf{N}}} + \underline{\underline{\mathbf{K}}} \right) \cdot \underline{\hat{\mathbf{T}}} = \underline{\mathbf{F}}$$

$$\underline{\underline{\mathbf{N}}} = \sum_{e=1}^{NE} \underline{\underline{\mathbf{N}}}^{G(e)} \quad ;$$

$$\underline{\underline{\mathbf{K}}} = \sum_{e=1}^{NE} \underline{\underline{\mathbf{K}}}^{G(e)} \quad ; \quad \underline{\mathbf{F}} = \sum_{e=1}^{NE} \underline{\mathbf{F}}^{G(e)}$$

The finite element method in heat transfer

$$K_{ij}^{G\,e} = \int_{\Omega^e} B_{im}\, k_{mp}\, B_{pj}\, d\Omega \qquad ; \qquad N_{ij}^{G\,e} = \int_{\Omega^e} h_i\, v_p\, B_{pj}\, d\Omega$$

$$\begin{bmatrix} \dfrac{\partial \tilde{T}}{\partial x} \\ \dfrac{\partial \tilde{T}}{\partial y} \\ \dfrac{\partial \tilde{T}}{\partial z} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial h_1}{\partial x} & \dfrac{\partial h_2}{\partial x} & \cdots & \dfrac{\partial h_n}{\partial x} \\ \dfrac{\partial h_1}{\partial y} & \dfrac{\partial h_2}{\partial y} & \cdots & \dfrac{\partial h_n}{\partial y} \\ \dfrac{\partial h_1}{\partial z} & \dfrac{\partial h_2}{\partial z} & \cdots & \dfrac{\partial h_n}{\partial z} \end{bmatrix} \cdot \underline{\hat{T}} = \underline{B} \cdot \underline{\hat{T}}$$

$$\underline{\underline{k}} = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$$

$$F_i^{G\,e} = \int_{\Omega^e} h_i\, q_v\, d\Omega - \int_{\Gamma_{q^e}} h_i\, q_{n_{imp}}$$

The Galerkin method is not good for

$$Pe \geq 1 \qquad ; \qquad Pe = \frac{vL^e}{2k}$$

The finite element method in heat transfer

$$[\nabla h_i] = \begin{bmatrix} \dfrac{\partial h_i}{\partial x} \\ \dfrac{\partial h_i}{\partial y} \\ \dfrac{\partial h_i}{\partial z} \end{bmatrix}$$

BUT $h_i = h_i(r,s,t)$

$$\begin{bmatrix} \dfrac{\partial(*)}{\partial r} \\ \dfrac{\partial(*)}{\partial s} \\ \dfrac{\partial(*)}{\partial t} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial x}{\partial r} & \dfrac{\partial y}{\partial r} & \dfrac{\partial z}{\partial r} \\ \dfrac{\partial x}{\partial s} & \dfrac{\partial y}{\partial s} & \dfrac{\partial z}{\partial s} \\ \dfrac{\partial x}{\partial t} & \dfrac{\partial y}{\partial t} & \dfrac{\partial z}{\partial t} \end{bmatrix} \begin{bmatrix} \dfrac{\partial(*)}{\partial x} \\ \dfrac{\partial(*)}{\partial y} \\ \dfrac{\partial(*)}{\partial z} \end{bmatrix} \quad ; \quad \underline{J} = \begin{bmatrix} \dfrac{\partial x}{\partial r} & \dfrac{\partial y}{\partial r} & \dfrac{\partial z}{\partial r} \\ \dfrac{\partial x}{\partial s} & \dfrac{\partial y}{\partial s} & \dfrac{\partial z}{\partial s} \\ \dfrac{\partial x}{\partial t} & \dfrac{\partial y}{\partial t} & \dfrac{\partial z}{\partial t} \end{bmatrix}$$

$$\begin{bmatrix} \dfrac{\partial(*)}{\partial x} \\ \dfrac{\partial(*)}{\partial y} \\ \dfrac{\partial(*)}{\partial z} \end{bmatrix} = \underline{J}^{-1} \begin{bmatrix} \dfrac{\partial(*)}{\partial r} \\ \dfrac{\partial(*)}{\partial s} \\ \dfrac{\partial(*)}{\partial t} \end{bmatrix}$$

The finite element method in heat transfer

$$\underline{H} = [h_1 \quad h_2 \quad \dots \quad h_{NNOD}] \quad ; \quad \underline{T} = \begin{bmatrix} T_1 \\ T_2 \\ \dots \\ T_{NNOD} \end{bmatrix} \quad ; \quad \tilde{T} = \underline{H}\,\underline{T}$$

$$\begin{bmatrix} \dfrac{\partial T}{\partial x} \\ \dfrac{\partial T}{\partial y} \\ \dfrac{\partial T}{\partial z} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial h_1}{\partial x} & \dfrac{\partial h_2}{\partial x} & \dots & \dfrac{\partial h_{NNOD}}{\partial x} \\ \dfrac{\partial h_1}{\partial y} & \dfrac{\partial h_2}{\partial y} & \dots & \dfrac{\partial h_{NNOD}}{\partial y} \\ \dfrac{\partial h_1}{\partial z} & \dfrac{\partial h_2}{\partial z} & \dots & \dfrac{\partial h_{NNOD}}{\partial z} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ \dots \\ T_{NNOD} \end{bmatrix}$$

$$\begin{bmatrix} \dfrac{\partial T}{\partial x} \\ \dfrac{\partial T}{\partial y} \\ \dfrac{\partial T}{\partial z} \end{bmatrix} = \underline{B}\,\underline{T}$$

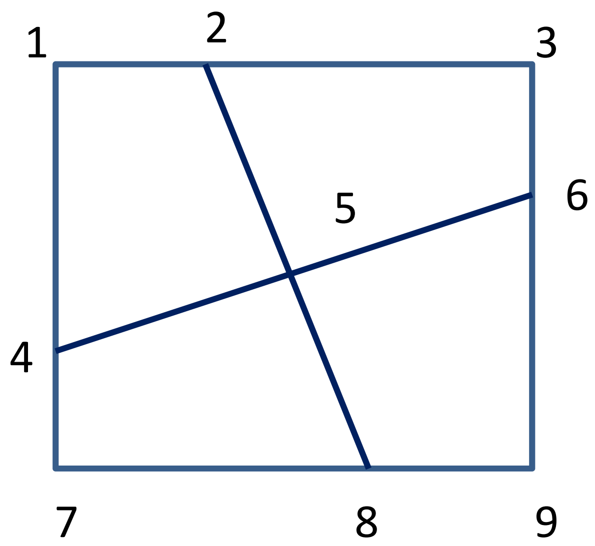
The finite element method in heat transfer

Notes:

- $\underline{J}$ must be non-singular; i.e. $\det(\underline{J}) \neq 0$

- We must choose $\det(\underline{J}) > 0 \; or \; \det(\underline{J}) > 0$

The finite element method in heat transfer



Elemental Matrix

$$\underline{K}^e = \begin{bmatrix} k^e{}_{11} & k^e{}_{12} & k^e{}_{13} & k^e{}_{14} \\ k^e{}_{21} & k^e{}_{22} & k^e{}_{23} & k^e{}_{24} \\ k^e{}_{31} & k^e{}_{32} & k^e{}_{33} & k^e{}_{34} \\ k^e{}_{41} & k^e{}_{42} & k^e{}_{43} & k^e{}_{44} \end{bmatrix}$$

$$\downarrow \sum_e$$

Global Matrix

Element 1

Local Node	Global Node
1	3
2	2
3	5
4	6

The finite element method in heat transfer

Exercises

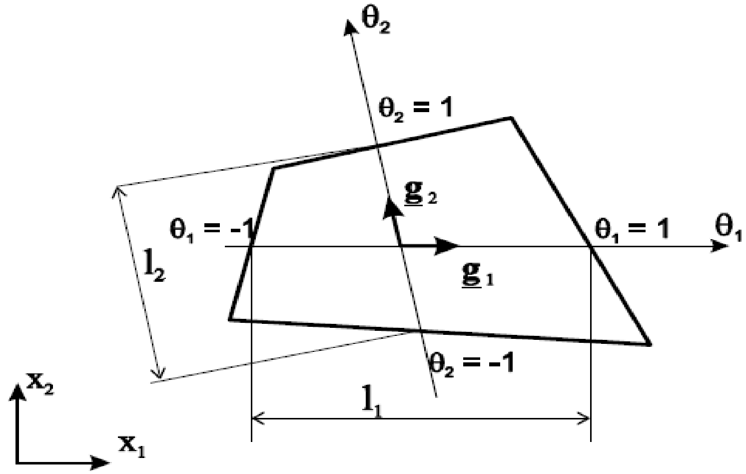
Local	Global
$K^{(1)}_{11}$	
$K^{(1)}_{12}$	
$K^{(1)}_{13}$	
$K^{(1)}_{14}$	
$K^{(1)}_{22}$	
$K^{(1)}_{23}$	
$K^{(1)}_{24}$	
$K^{(1)}_{33}$	
$K^{(1)}_{34}$	
$K^{(1)}_{44}$	

Local	Global
$K^{(2)}_{11}$	
$K^{(2)}_{34}$	
$K^{(3)}_{13}$	
$K^{(3)}_{44}$	
$K^{(4)}_{22}$	
$K^{(4)}_{23}$	

The finite element method in heat transfer

Matrix	Dimensions	Observations
C	NNOD x NNOD	Sym.
T	NNOD x 1	
N	NNOD x NNOD	Non-sym N=0 In Lagrangian formulation
K	NNOD x NNOD	Sym
Q	NNOD x 1	

The finite element method in heat transfer



SUPG: Streamline Upwind Petrov Galerkin Method : The weighted functions are different of the approximation functions

$$W_i = \tau \, \underline{\mathbf{v}} \cdot \underline{\nabla} h_i \quad ; \quad \tau = \sum_{i=1}^{ND} \frac{\alpha_i \, v_i^c \, l_i}{2} \frac{1}{||\underline{\mathbf{v}}||^2} \quad ;$$

$$\alpha_i = \coth |Pe_i| - \frac{1}{|Pe_i|} \quad ; \quad Pe_i = \frac{v_i \, l_i}{2 \, k}$$

$$v_i^c = \underline{\mathbf{g}}_i \cdot \underline{\mathbf{v}}^c \quad ; \quad \underline{\mathbf{v}}^c = \text{is the central velocity} \;\; ;$$

$$ND = \text{number of dimension} \;\; ; \;\; l_i = \text{characteristics dimension}$$

$$||\underline{\mathbf{v}}|| = \left[\sum_{i=1}^{ND} v_i^2 \right]^{1/2} \qquad \underline{\mathbf{H}}^T = (h_1, h_2, ..., h_{nnodo})$$

The finite element method in heat transfer

SUPG: Streamline Upwind Petrov Galerkin Method

$$\underline{\underline{\mathbf{M}}} \cdot \dot{\hat{\underline{\mathbf{T}}}} + \left(\underline{\underline{\mathbf{N}}} + \underline{\underline{\mathbf{K}}} \right) \cdot \hat{\underline{\mathbf{T}}} = \underline{\mathbf{F}}$$

$$\underline{\underline{\mathbf{M}}} = \sum_{e=1}^{NE} \left(\underline{\underline{\mathbf{M}}}^{G^{(e)}} + \underline{\underline{\mathbf{M}}}^{P^{(e)}} \right) \quad ; \quad \underline{\underline{\mathbf{N}}} = \sum_{e=1}^{NE} \left(\underline{\underline{\mathbf{N}}}^{G^{(e)}} + \underline{\underline{\mathbf{N}}}^{P^{(e)}} \right)$$

$$\underline{\underline{\mathbf{K}}} = \sum_{e=1}^{NE} \left(\underline{\underline{\mathbf{K}}}^{G^{(e)}} + \underline{\underline{\mathbf{K}}}^{P^{(e)}} \right) \quad ; \quad \underline{\mathbf{F}} = \sum_{e=1}^{NE} \left(\underline{\mathbf{F}}^{G^{(e)}} + \underline{\mathbf{F}}^{P^{(e)}} \right)$$

The finite element method in heat transfer

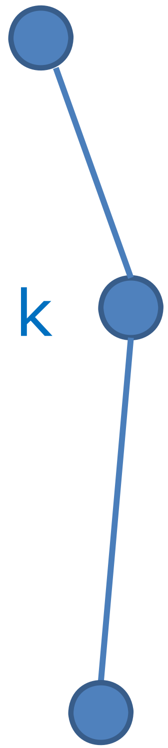
SUPG: Streamline Upwind Petrov Galerkin Method

$$N_{ij}^{P^{(e)}} = \int_{\Omega^e} W_i \, \rho C_p \underline{\mathbf{v}} \cdot \underline{\boldsymbol{\nabla}} h_j \, d\Omega$$

$$K_{ij}^{P^{(e)}} = - \int_{\Omega^e} W_i \, \underline{\boldsymbol{\nabla}} \cdot \left(\underline{\underline{\mathbf{k}}} \cdot \underline{\boldsymbol{\nabla}} h_j \right) \, d\Omega$$

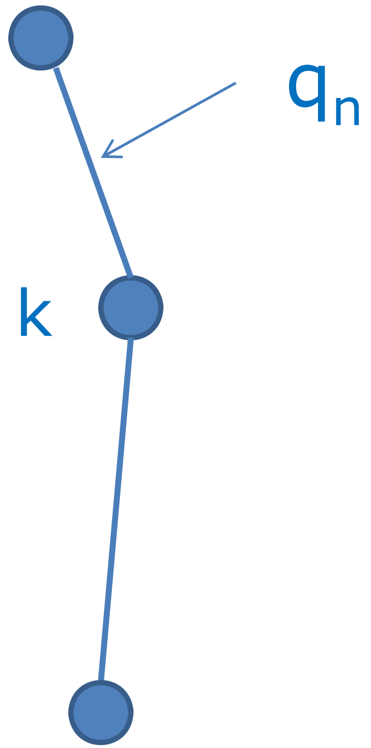
$$F_i^{P^{(e)}} = \int_{\Omega^e} W_i \, q_v \, d\Omega$$

Boundary conditions: Dirichlet

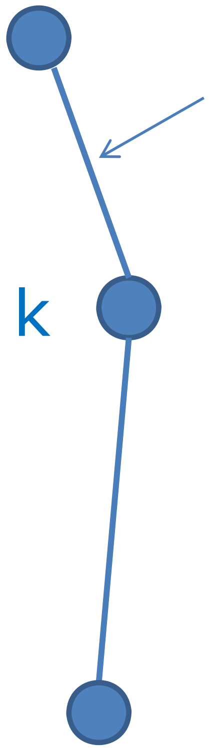


T_k=fixed

Boundary conditions: natural



Boundary conditions: mixed



$q_n = h(T - T_{environment})$

k

Normally h=h(T)
(nonlinear)

Boundary conditions

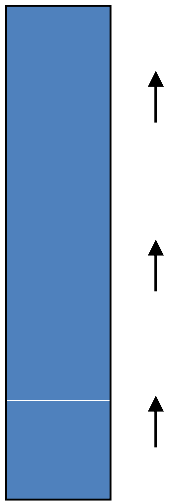
- Dirichlet boundary conditions $T = T^{\text{sup}} \qquad \forall(\underline{x}, t) \in \partial\Gamma_T \times \partial t$
- Newmann boundary conditions $q_n^* = -k\,\frac{\partial T}{\partial n} = Q_s(T) \qquad \forall(\underline{x}, t)$ $\partial\Gamma_q \times \partial t$
  - where $\underline{\mathbf{n}}$ is the normal vector output to the domain surface Q_s is positive when the heat output Ω.
- Mixt boundary conditions. Newton cooling law

$$q_n^* = -k\,\frac{\partial T}{\partial n} = h(T)\,(T - T_{amb}) \qquad \forall(\underline{x}, t) \in \partial\Gamma_c \times \partial t$$

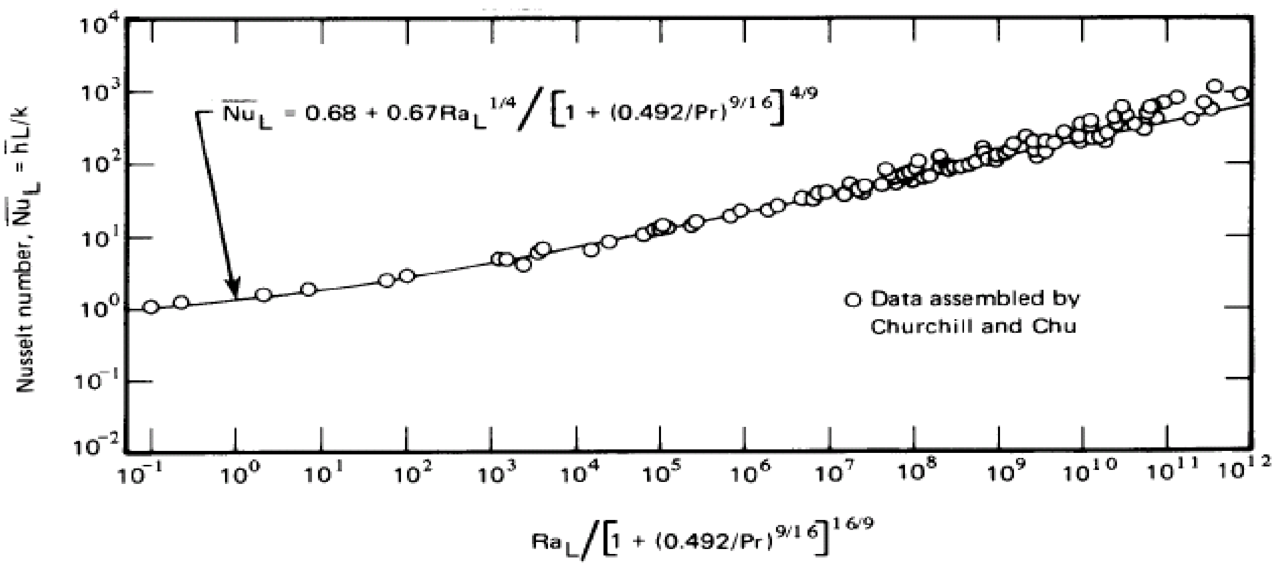
Boundary conditions: Natural convection

Vertical surface

$$Nu = 0.68 + \frac{0.67\ Ra^{1/4}}{\left[1 + \left(0.492\ /\ \Pr\right)^{9/16}\right]^{4/9}}$$



$T_{wall} > T_{fluid}$

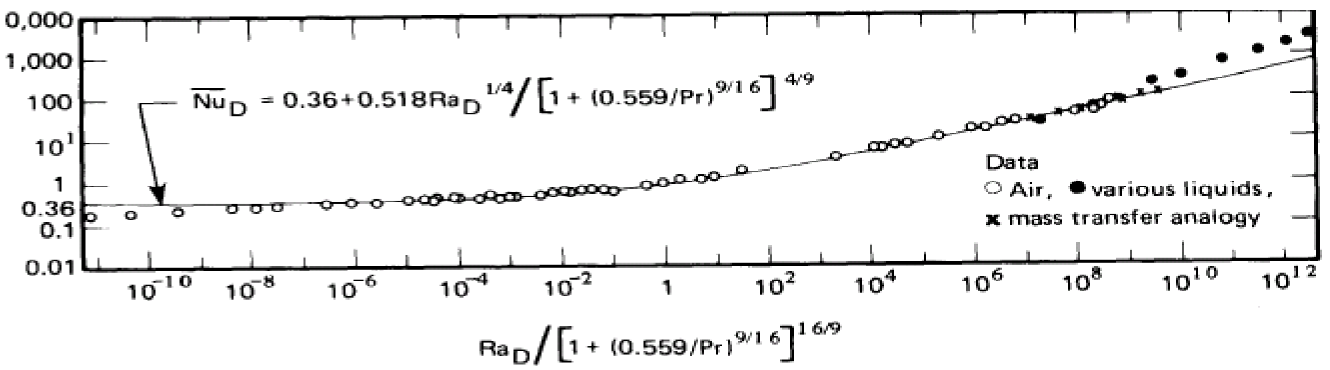


Boundary conditions: Natural convection

Horizontal cylinder

$$Nu = \left\{ 0.60 + \frac{0.387\ Ra^{1/6}}{\left[1 + \left(0.559 / \mathrm{Pr} \right)^{9/16} \right]^{8/27}} \right\}^{2} \qquad Ra \geq 10^{-6}$$

$$Nu = 0.36 + \frac{0.518\ Ra^{1/4}}{\left[1 + \left(0.559 / \mathrm{Pr} \right)^{9/16} \right]^{4/9}} \qquad 10^{-6} \leq Ra \leq 10^{9}$$

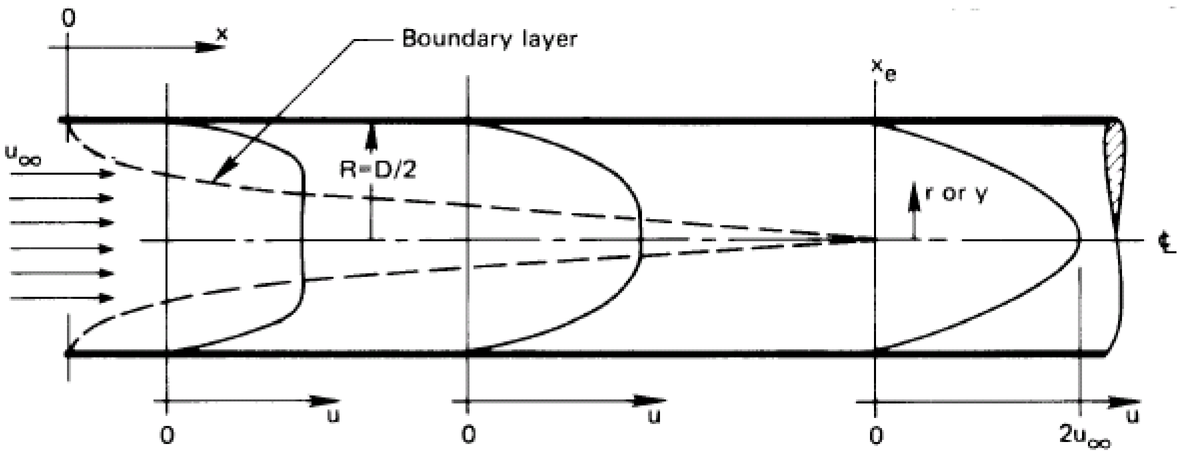


Boundary conditions: Natural convection

$$Nu = \left\{ A_1 + \frac{A_2 \, Ra^{n_1}}{\left[1 + \left(A_3 / \Pr \right)^{n_2} \right]^{n_3}} \right\}^{n_4}$$

<table>
<tr><th></th><th>A_1</th><th>A_2</th><th>A_3</th><th>n_1</th><th>n_2</th><th>n_3</th><th>n_4</th></tr>
<tr><td>Laminar vertical plate</td><td>0.68</td><td>0.670</td><td rowspan="2">0.492</td><td>1/4</td><td rowspan="4">9/16</td><td>4/9</td><td>1</td></tr>
<tr><td>Turbulent vertical plate</td><td>0.825</td><td>0.387</td><td>1/6</td><td>8/27</td><td>2</td></tr>
<tr><td>Laminar horizontal cylinder</td><td>0.36</td><td>0.518</td><td rowspan="2">0.559</td><td>1/4</td><td>4/9</td><td>1</td></tr>
<tr><td>Turbulent horizontal cylinder</td><td>0.6</td><td>0.387</td><td>1/6</td><td>8/27</td><td>2</td></tr>
</table>

Boundary conditions: Forced convection



Graezt number correlates the influence of entrance effects of a pipe

$$Gz = \mathrm{Re}\,\mathrm{Pr}\,\frac{D}{L}$$

Boundary conditions: Forced convection

Laminar flow in a tube

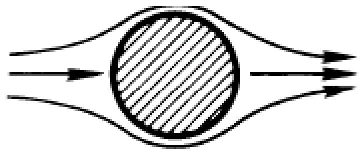
$$Nu = 3.66 + \frac{0.0668\ Gz}{1 + 0.04\ Gz^{2/3}}$$

Turbulent flow in a tube

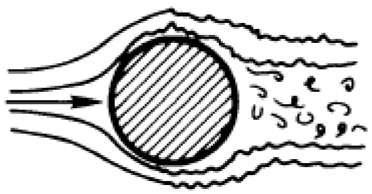
$$Nu = 0.023\ \mathrm{Re}^{0.8}\ \mathrm{Pr}^{1/3}$$

Boundary conditions: Forced convection

Flow across a body



Laminar



urbulent

$$Nu = C\,\mathrm{Re}^n\,\mathrm{Pr}^{1/3}$$

TABLE 7-5-1
Values of C and n for Flow over Circular Cylinders

Re	C	n
0.4–4	0.919	0.330
4–40	0.911	0.385
40–4000	0.683	0.466
4,000–40,000	0.193	0.618
40,000–400,000	0.0266	0.805

TABLE 7-5-2
Values of C and n for Flow over Noncircular Cylinders

Geometry	Flow incidence	Re range	C	n
Square	Corner	5×10^3–10^5	0.246	0.588
Square	Side	5×10^3–10^5	0.102	0.675
Hexagon	Corner	5×10^3–10^5	0.153	0.638
Hexagon	Side	5×10^5–1.95×10^4	0.160	0.650
		1.95×10^4–10^5	0.0385	0.782
Vertical plate	Normal	4×10^3–1.5×10^4	0.228	0.731

Becker, Heat Transfer

Boundary conditions: radiation

- Radiation boundary conditions

$$q_n^* = -k \frac{\partial T}{\partial n} = \sigma F \epsilon \left(T^4 - T^4_{medio} \right) \qquad \forall (\underline{x}, t) \in \partial\Gamma_r \times \partial t$$

where σ is the Stefan-Boltzmann constant $= 5.6697 \times 10^{-8} \frac{\mathrm{W}}{\mathrm{m}^2 \cdot \mathrm{K}^4}$;
ϵ is the emisitivy (nondimensional), F is the shape factor or vision factor (nondimensional)

- Robin boundary conditions

$$q_n^* = -k \frac{\partial T}{\partial n} = \hat{h}(T) \left(T - T_{amb} \right) \qquad \forall (\underline{x}, t) \in \partial\Gamma_\sigma \times \partial t$$

$$\begin{aligned} \hat{h} &= \hat{h}_{conv} + \hat{h}_{rad} \\ \hat{h}_{conv} &= \textit{experimental, literature, etc.} \\ \hat{h}_{rad} &= \sigma F \epsilon \frac{T^4 - T^4_{medio}}{T - T_{amb}} \end{aligned}$$

Boundary conditions: radiation

Table 10.1 Total emittances for a variety of surfaces [10.1]

Metals			*Nonmetals*		
Surface	*Temp.* (°C)	ε	*Surface*	*Temp.* (°C)	ε
Aluminum			Asbestos	40	0.93–0.97
Polished, 98% pure	200–600	0.04–0.06	Brick		
Commercial sheet	90	0.09	Red, rough	40	0.93
Heavily oxidized	90–540	0.20–0.33	Silica	980	0.80–0.85
Brass			Fireclay	980	0.75
Highly polished	260	0.03	Ordinary refractory	1090	0.59
Dull plate	40–260	0.22	Magnesite refractory	980	0.38
Oxidized	40–260	0.46–0.56	White refractory	1090	0.29
Copper			Carbon		
Highly polished electrolytic	90	0.02	Filament	1040–1430	0.53
Slightly polished to dull	40	0.12–0.15	Lampsoot	40	0.95
Black oxidized	40	0.76	Concrete, rough	40	0.94
Gold: pure, polished	90–600	0.02–0.035	Glass		
Iron and steel			Smooth	40	0.94
Mild steel, polished	150–480	0.14–0.32	Quartz glass (2 mm)	260–540	0.96–0.66
Steel, polished	40–260	0.07–0.10	Pyrex	260–540	0.94–0.74
Sheet steel, rolled	40	0.66	Gypsum	40	0.80–0.90
Sheet steel, strong rough oxide	40	0.80	Ice	0	0.97–0.98
Cast iron, oxidized	40–260	0.57–0.66	Limestone	400–260	0.95–0.83
Iron, rusted	40	0.61–0.85	Marble	40	0.93–0.95
Wrought iron, smooth	40	0.35	Mica	40	0.75
Wrought iron, dull oxidized	20–360	0.94	Paints		
Stainless, polished	40	0.07–0.17	Black gloss	40	0.90
Stainless, after repeated heating	230–900	0.50–0.70	White paint	40	0.89–0.97
			Lacquer	40	0.80–0.95
Lead			Various oil paints	40	0.92–0.96
Polished	40–260	0.05–0.08	Red lead	90	0.93
Oxidized	40–200	0.63	Paper		
Mercury: pure, clean	40–90	0.10–0.12	White	40	0.95–0.98
Platinum			Other colors	40	0.92–0.94
Pure, polished plate	200–590	0.05–0.10	Roofing	40	0.91
Oxidized at 590°C	260–590	0.07–0.11	Plaster, rough lime	40–260	0.92
Drawn wire and strips	40–1370	0.04–0.19	Quartz	100–1000	0.89–0.58
Silver	200	0.01–0.04	Rubber	40	0.86–0.94
Tin	40–90	0.05	Snow	10–20	0.82
Tungsten			Water, thickness ≥0.1 mm	40	0.96
Filament	540–1090	0.11–0.16	Wood	40	0.80–0.90
Filament	2760	0.39	Oak, planed	20	0.90

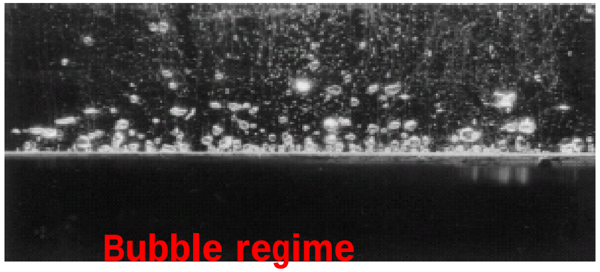
Boundary conditions: boiling

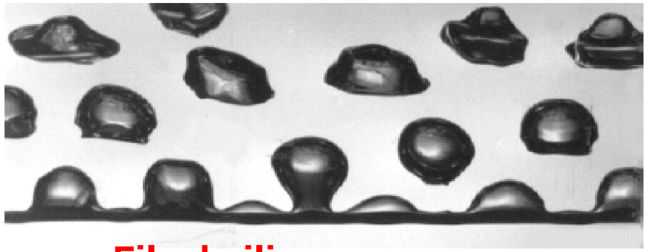
When evaporation occurs at a solid-liquid interface, it is named boiling.
This process is characterized by the formation of vapor bubbles which
grow and subsequently detach from the surface.

The process occurs when the temperature of the solid surface exceeds the saturation temperature T_{sat} corresponding to the liquid pressure.

The heat transfer convection coefficient (h) and the heat flux (q) depend not only on the vapor and liquid properties at saturation temperature but also on the excess temperature (DTe)
DTe = Ts - Tsat (1)

Nukiyama (1934) analized the boiled saturated water on a horizontal wire with an electric resistance heater





Film boiling

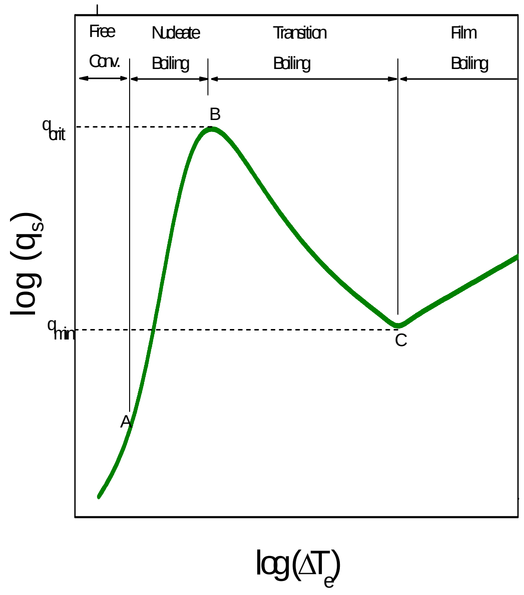
Boundary conditions: boiling

The boiling curve (heat flux as a function of excess temperature is usually presented at a liquid temperature equal to saturation temperature sat T , corresponding to the liquid pressure.

In pool boiling the liquid is quiescent and its motion near the steel surface is due to free convection. In contrast, for forced convection boiling, fluid motion is induced by external means, as well as by free convection and bubble-induced mixing.

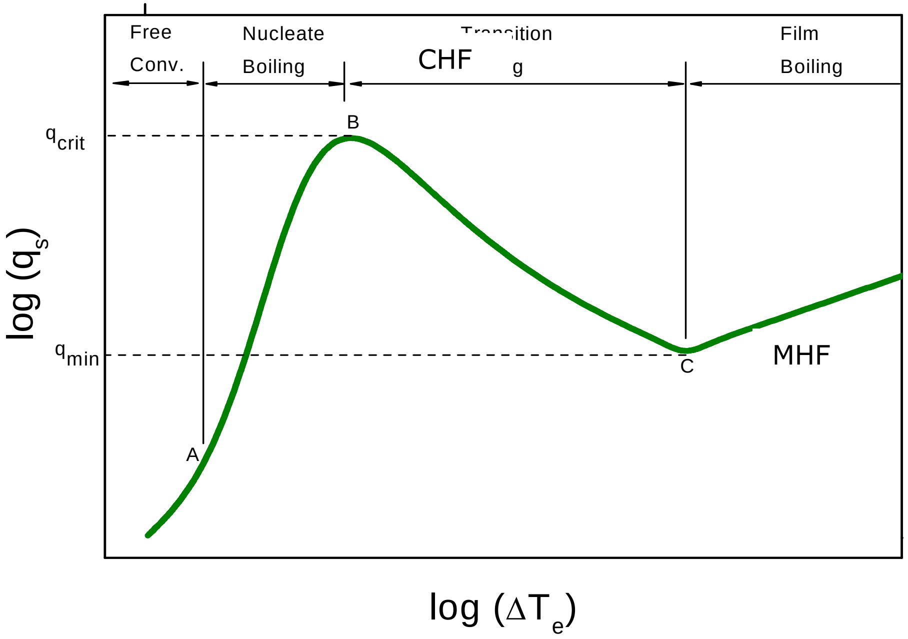
Boiling may also be classified according to whether it is subcooled or saturated. In subcooled boiling, the temperature of the liquid is below the saturation temperature and bubbles formed at the surface may condense in the liquid.

Both, forced convection and subcooling are known to increase the critical and minimum heat fluxes (CHF and MHF, respectively). The film boiling heat transfer coefficient increases with velocity and subcooling, and it is further elevated by radiation effects at higher temperatures.



Boundary conditions: boiling

Heat is transferred from the solid surface to the liquid: $q = h\left(T_s - T_{sat}\right) = h\Delta T_e$



1) **Free convection**: There is insufficient vapor in contact with the liquid phase to cause boiling
2) **Nucleate boiling**: isolated bubbles form at nucleation sites and separate from the surface or the vapor escapes as jets or columns.
3) **Transition boiling**: Bubble formation is so rapid that a vapor film begins to form on the surface.
4) **Film boiling**: the surface is completely covered by a vapor blanket.

Boundary conditions: boiling

Typical correlations for the various boiling modes or flow regimes in pool boiling and forced convection subcooled boiling have been developed by:

1. F. P. Incropera and D. P. DeWitt, *Fundamentals of Heat and Mass Transfer,* 4th ed.,John Wiley & Sons, New York, 1996.

2. A. F. Mills, *Heat Transfer*, Irwin, Illinois, 1992.

3. R. T. Lahey, *Boiling Heat Transfer,* New York, 1992.

4. S. Yilmaz, J. W. Westwater, *Effect of velocity of heat transfer to boiling freon -113,* J. Heat Transfer, **102** (1980) 26.

In the following slides the heat flow rate equations of different boiling mechanisms are shown, and a validation with a Jominy test.

Boundary conditions: boiling

1) Free convection: $T_s < T_{sat} + 5°C$ Water T_{sat} = 100 ° C

$$q = \frac{k_l}{D\left(T_s - T_{sat}\right)}\left\{0.6 + \frac{0.387\ Ra_D^{1/6}}{\left[1 + \left(0.559 / \Pr\right)^{9/16}\right]^{8/27}}\right\}^2$$

2) Nucleate boiling: from $dT_e > 5°C$ to critical flux

$$q = \frac{k_l}{\left(T_s - T_{sat}\right)}\left(\frac{\left(\rho_l - \rho_v\right)g}{\sigma}\right)^{1/2}\left(\frac{c_{pl}\left(T_s - T_{sat}\right)}{h_{fg}}\right)^2 \frac{1}{C_{nb}^3 \Pr_l^m}$$

Critical Heat flux (CHF)

$$q_{crit} = 0.149\ \rho_v h_{fg}\left[\frac{g\left(\rho_l - \rho_v\right)\sigma}{\rho_v^2}\right]^{1/4}$$

Boundary conditions: boiling

Minimum Heat flux (MHF, Leidenfrost point)

$$q_{\min} = C \rho_v h_{fg} \left[\frac{g \sigma (\rho_l - \rho_v)}{(\rho_l + \rho_v)^2} \right]^{1/4}$$

3) Film boiling

$$q = C_{film} \left\{ \frac{g (\rho_l - \rho_v) h'_{fg} k_v^3 \rho_v (T_s - T_{sat})^3}{\mu_v D} \right\}^{1/4}$$

Radiation: The temperature level is generally quite high in film boiling, and it is necessary to account for radiation. A simple superposition of heat transfer coefficient is not adequate. The following equation is recommended:

$$h_T = h \left(\frac{h}{h_T} \right)^{1/3} + h_r \approx h + 0.75\, h_r$$

Boundary conditions: boiling

cp_l : Liquid specific heat $[J/KgK]$

cp_v : Vapor specific heat $[J/KgK]$

D_J : Nozzle diameter (column of water) $[m]$

D_M : Specimen diameter $[m]$

g : Gravity acceleration $[m/s^2]$

h : Heat transfer convection coefficient $[W/m^2K]$

h_{CF} : Heat transfer coefficient for forced convection $[W/m^2K]$

h_{fg} : Latent heat of vaporization $[J/Kg]$

H_J : Distance between the bottom of specimen and the opening of the water nozzle $[m]$

H_L : Free height of the column of water $[m]$

H_M : Specimen length $[m]$

h_r : Radiation heat transfer coefficient $[W/m^2K]$

k_l : Liquid thermal conductivity $[W/mK]$

k_v : Vapor thermal conductivity $[W/mK]$

Nu_D : Nusselt number; $Nu_D = \frac{hD}{k_l}$ (dimensionless)

$\Pr_l$: Liquid Prandl number; $\Pr_l = \frac{cp_l \rho_l}{k_l}$ (dimensionless)

q : Heat flux $[W/m^2]$

q_B : Boiling heat flux $[W/m^2]$

q_{CF} : Heat flux due forced convection $[W/m^2]$

Boundary conditions: boiling

q_{crit} : Critical heat flux $[W/m^2]$

$q_{crit,sat}$: Critical heat flux at saturated temperature $[W/m^2]$

$q_{crit,sub}$: Critical heat flux due subcooling $[W/m^2]$

$q_{\min}$: Minimum heat flux $[W/m^2]$

Ra_D : Rayleigh number; $Ra_D = \dfrac{g\beta cp_l (T_s - T_\infty) D^3}{\mu_l k_l}$ (dimensionless)

Re_D : Reynolds number; $\mathrm{Re}_D = \dfrac{\rho_l V_m D}{\mu_l}$ (dimensionless)

T_{bulk} : Bulk fluid temperature $[K]$

T_s : Temperature of heated surface $[K]$

T_{sat} : Saturation liquid temperature $[K]$

V_m : Internal flow velocity $[m/s]$

ΔT_e : Excess temperature; $\Delta T_e = T_s - T_{sat}$ $[K]$

μ_l : Liquid viscosity $[Ns/m^2]$

μ_v : Vapor viscosity $[Ns/m^2]$

ρ_l : Liquid density $[Kg/m^3]$

ρ_v : Vapor density $[Kg/m^3]$

σ : Surface tension $[N/m]$

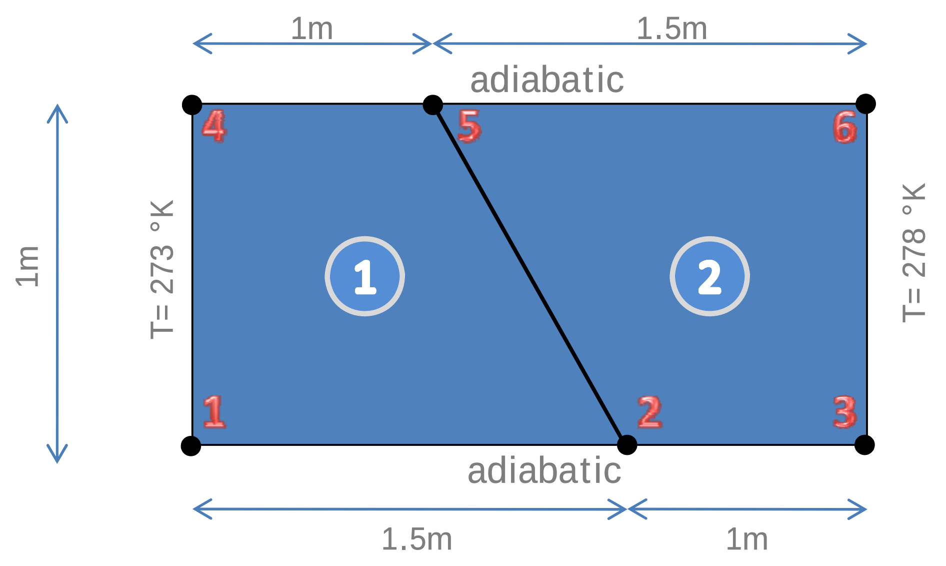
σ_{SB} : Stefan-Boltzmann constant $[W/m^2K^4]$

EXERCISES

Exercise 1: Obtain the FEA formulation for Linear Stationary heat transfer with convection.

Exercise 2: Calculate the Nodal Temperature and the heat flux on the gauss points for the following case with the prescripted discretization shown below. K=440 W/(m.ºK)

(a)



Examples on conduction –convection heat transfer problems

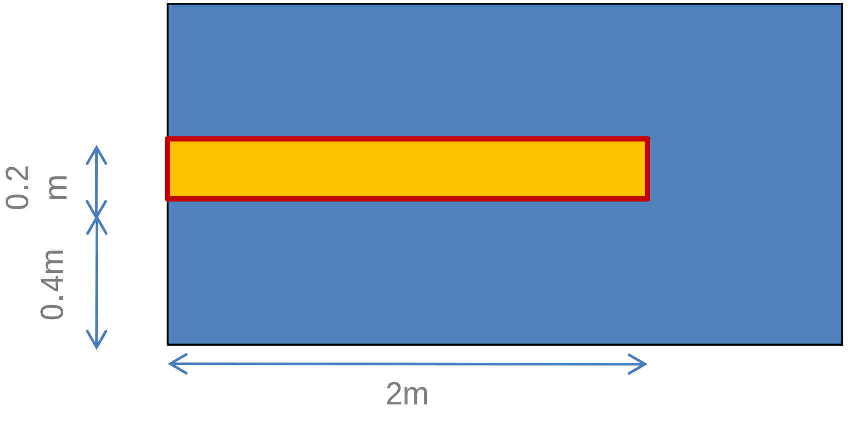
(b) Repeat the exercise without imposing the adiabatic condition.

(c) Change the adiabatic condition with a prescribed convection load considering an environment temperature of 300ºK and a convection coefficient H=100 W/(m^2.ºK)

(d) Increase H untill H=1e7. Extract your own conclusions about the results obtained.

(d) Repeat exercise (c) remeshing with an arbitrary structural dense mesh. Is there any difference in the results?

(e) Include an internal heat electrode with an internal heat load of 2e5 W/m^3 as shown below



Examples on conduction –convection heat transfer problems

Exercise 3: Consider a 90º semi-infinite cylinder. Side BC is subjected to prescribed temperature of 50º. Side AB is the symmetry axis (axilsymmetryc). Calculate the temperature profile.