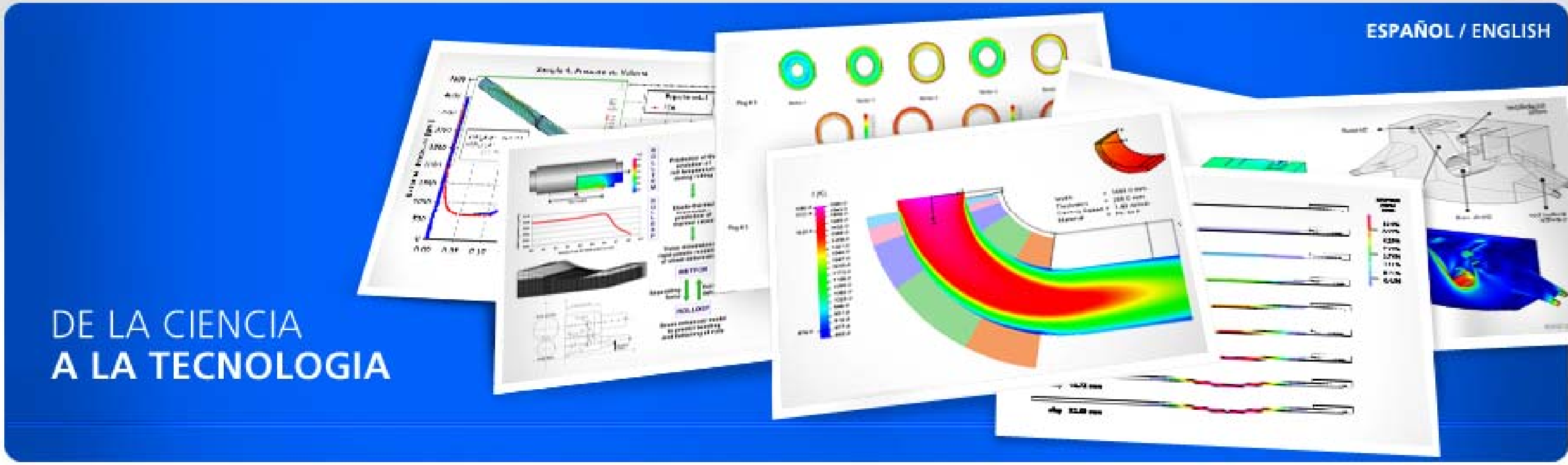
SIM&TEC

Simulación y Tecnología

Simulation and Technology

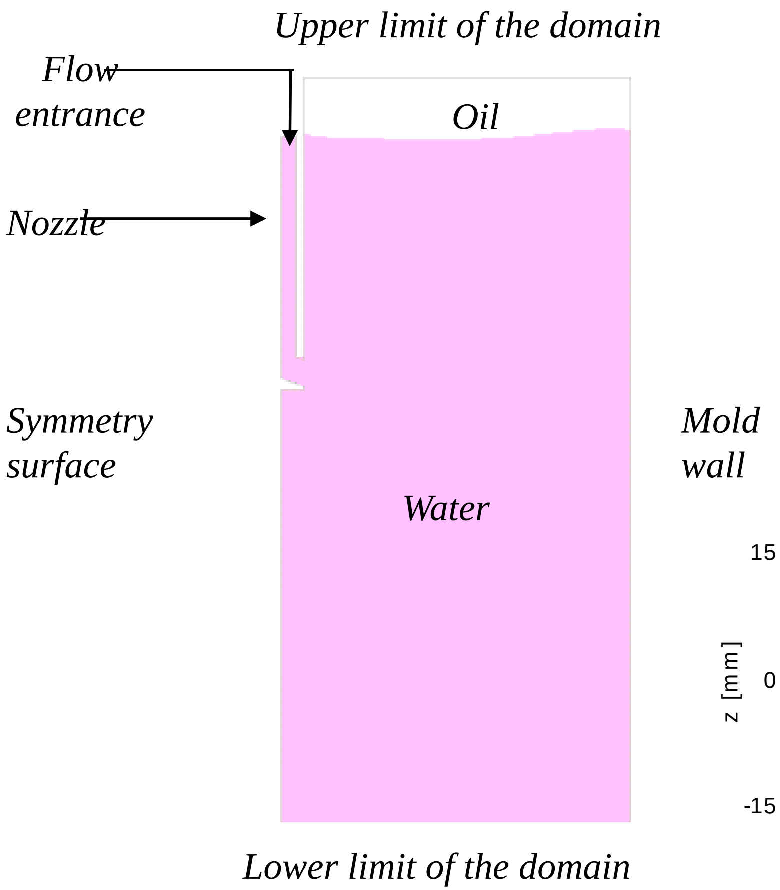


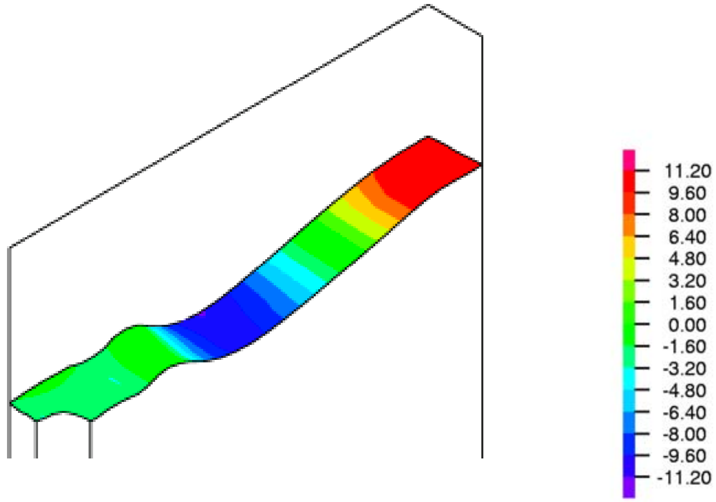
FLUID DYNAMICS

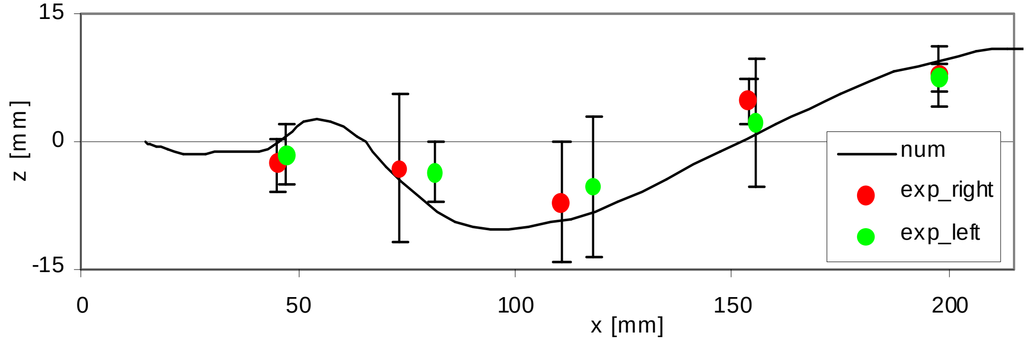
Part 3: Interphases

Marcela B. Goldschmit

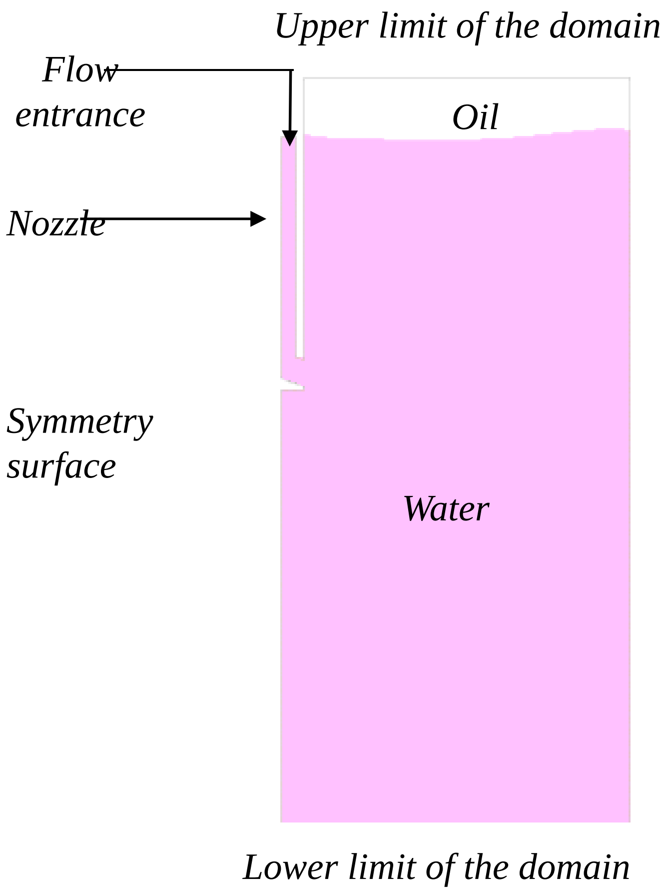
Interphases

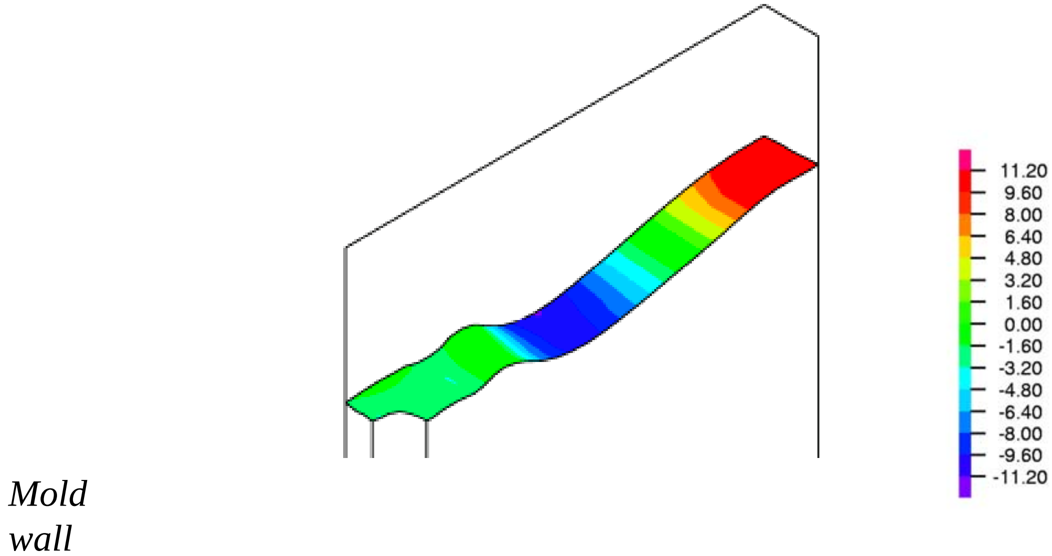


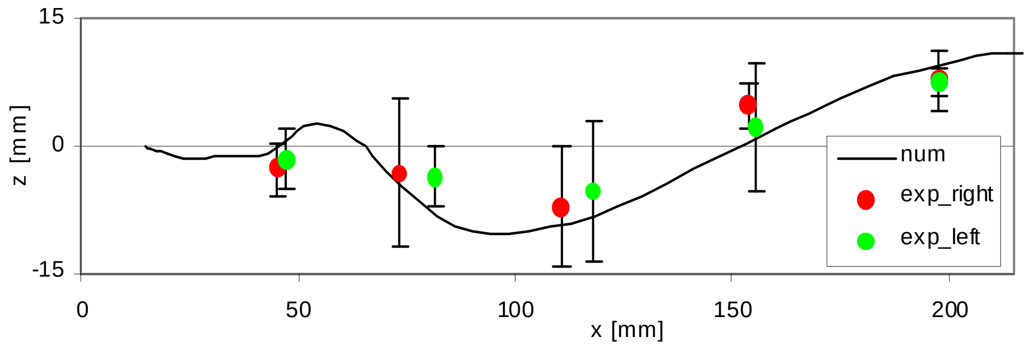




Interphases

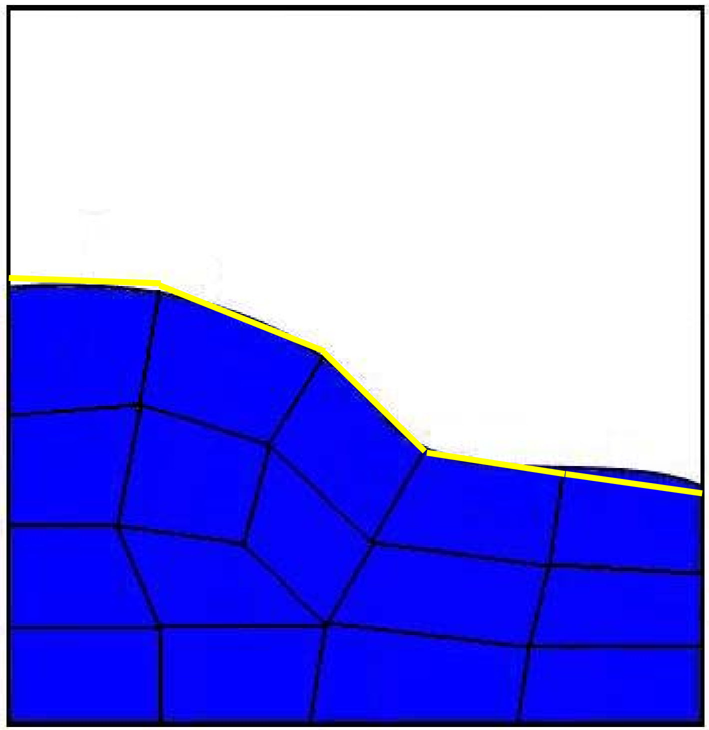




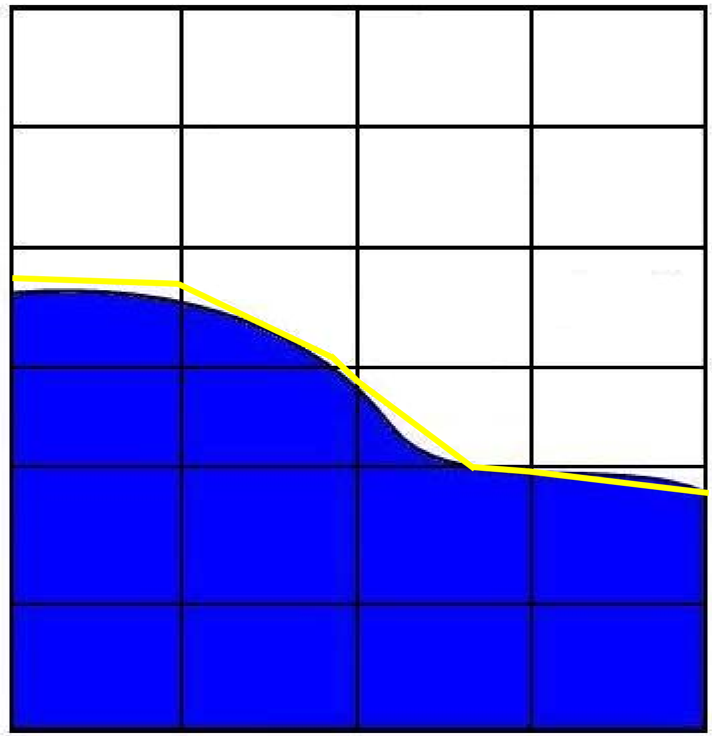


Interphases

Lagrangian Method



Método Euleriano



Interphases

- Define:

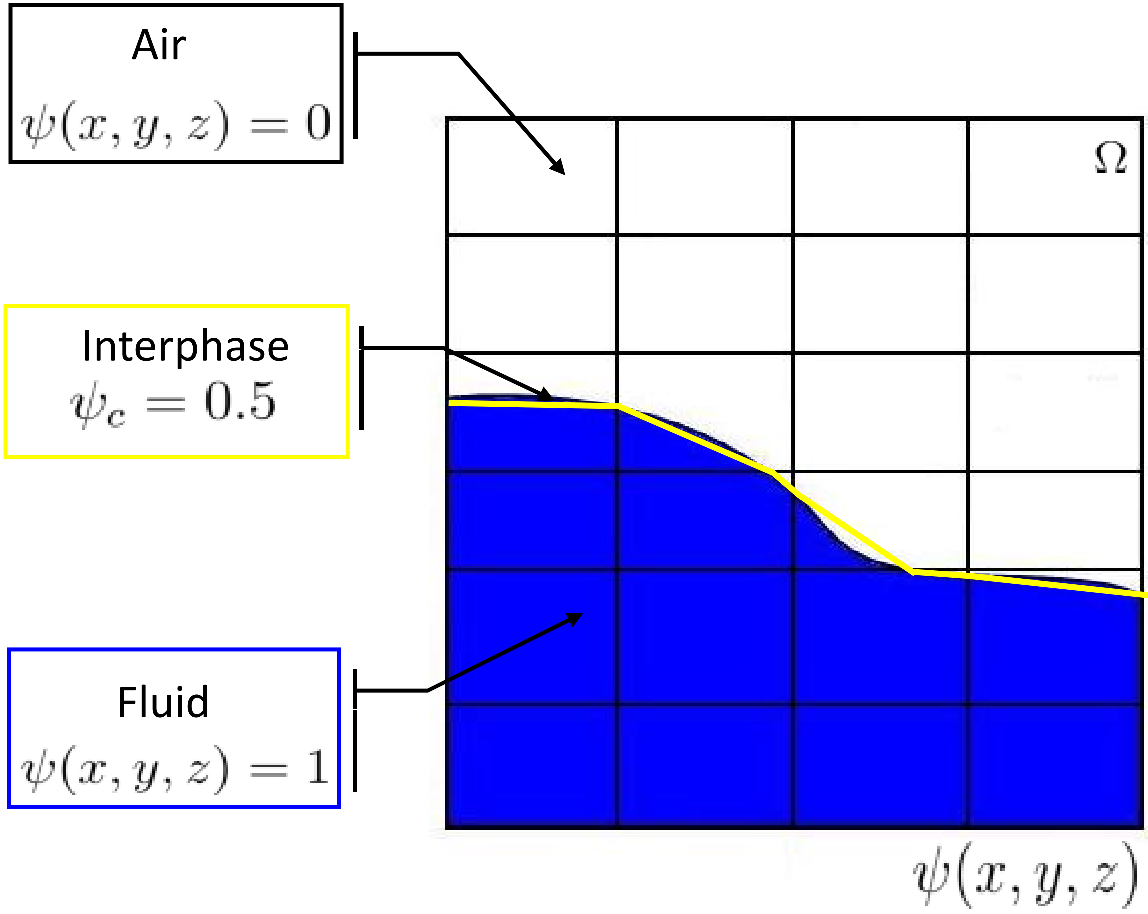
$$\psi(x, y, z) \in \Omega$$

- Limits

$$0 \leq \psi(x, y, z) \leq 1$$

- Define:

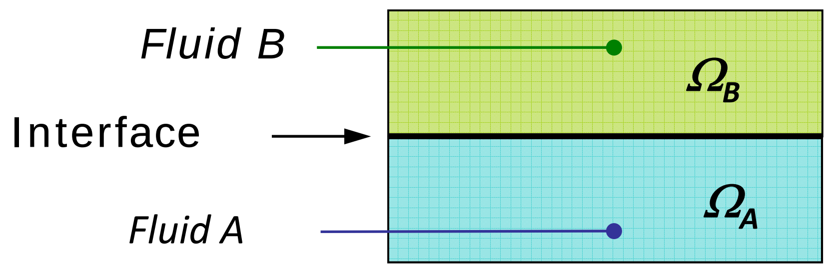
$$\psi(x, y, z) = \psi_c$$



Interphases

Pseudoconcentration

$$\frac{\partial \psi_{(\underline{x},t)}}{\partial t} + \underline{v} \cdot \nabla \psi_{(\underline{x},t)} = 0$$



Critical value C_c :

$\Psi(\underline{x},t) > \Psi_C$ For $\underline{x}$ in Ω_A.

$\Psi(\underline{x},t) < \Psi_C$ For $\underline{x}$ in Ω_A.

$\Psi(\underline{x},t) = \Psi_C$ For $\underline{x}$ on the Interface.

$$\psi = \psi_c + \sigma \; d \; signo(\psi_{\underline{x}} - \psi_c)$$

d($\underline{x}$) is the distance from the node located at $\underline{x}$ to the interface.

Interphases

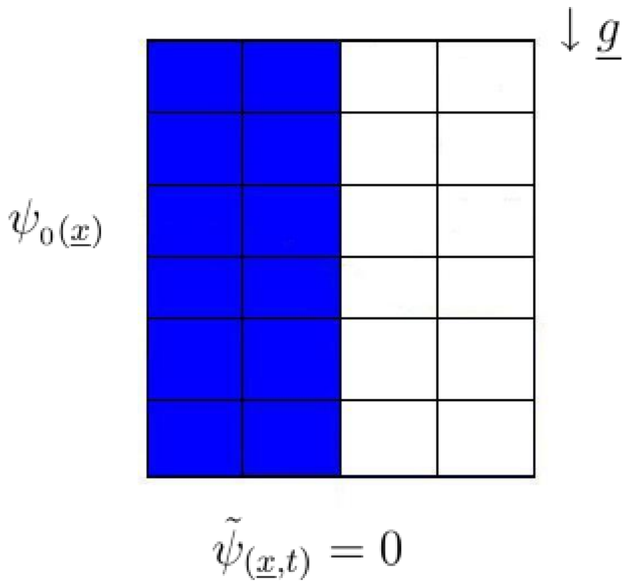
Condiciones de Contorno

$$\psi_{(\underline{x},t)} = \tilde{\psi}_{(\underline{x},t)} \quad \forall \underline{x} \in \Gamma_c : \{ \underline{v} \cdot \underline{\hat{n}} < 0 \}$$

Esto se debe a que la ecuación es hiperbólica, requiere únicamente condiciones de contorno donde el flujo es entrante.

Condiciones Iniciales

$$\psi_{(\underline{x},0)} = \psi_{0(\underline{x})}$$



Interphases

FEM – Theta Method - SUPG Method

$$ {}^{t+\Delta t}\hat{\psi}^{j} = (1-\theta)\, {}^{t}\hat{\psi}^{j} + \theta\, {}^{t+\Delta t}\hat{\psi}^{j} $$

$$ {}^{t+\Delta t}\varphi + \theta\,\Delta t\left({}^{t+\Delta t}\upsilon_k \frac{\partial\, {}^{t+\Delta t}\varphi}{\partial x_k} \right) = {}^{t}\varphi - (1-\theta)\left({}^{t}\upsilon_k \frac{\partial\, {}^{t}\varphi}{\partial x_k} \right) $$

$$ \sum_e \int_\Omega \left(h^i + \omega^i\right)\left[{}^{t+\Delta t}\varphi + \theta\,\Delta t\left({}^{t+\Delta t}\upsilon_k \frac{\partial\, {}^{t+\Delta t}\varphi}{\partial x_k} \right)\right] d\Omega = \sum_e \int_\Omega \left(h^i + \omega^i\right)\left[{}^{t}\varphi - (1-\theta)\left({}^{t}\upsilon_k \frac{\partial\, {}^{t}\varphi}{\partial x_k} \right)\right] d\Omega $$

Interphases

FEM – Theta Method - SUPG Method

$$^{t+\Delta t}\psi = h^j \; ^{t+\Delta t}\psi^j$$

$$\sum_e \int_\Omega \left(h^i + \omega^i\right)\left[h^j + \theta\,\Delta t\left({}^{t+\Delta t}v_k \frac{\partial h^j}{\partial x_k}\right)\right] d\Omega \; {}^{t+\Delta t}\hat{\varphi}^j = \sum_e \int_\Omega \left(h^i + \omega^i\right)\left[{}^t\varphi - (1-\theta)\left({}^t v_k \frac{\partial\, {}^t\varphi}{\partial x_k}\right)\right] d\Omega$$

$$\underline{\underline{M}} \cdot \underline{\hat{\varphi}} = \underline{F}$$

Interphases

FEM – Theta Method - SUPG Method

$$h^i + w^i = h^i + \tau \; v^c \; \partial_c h^i$$

$$\tau = \sum_{n=1}^{nd} \frac{\beta_i v_i l_i}{2 \mid \underline{v_c} \mid^2} \qquad \beta_i = coth(Pe_i) - \frac{1}{Pe_i} \qquad Pe_i = \frac{\mid v_i \mid l_i}{2D}$$

$$\mid v_c \mid = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

Interphases

$$\psi = \psi_c + \sigma\ d\ signo(\psi_{\underline{x}} - \psi_c)$$

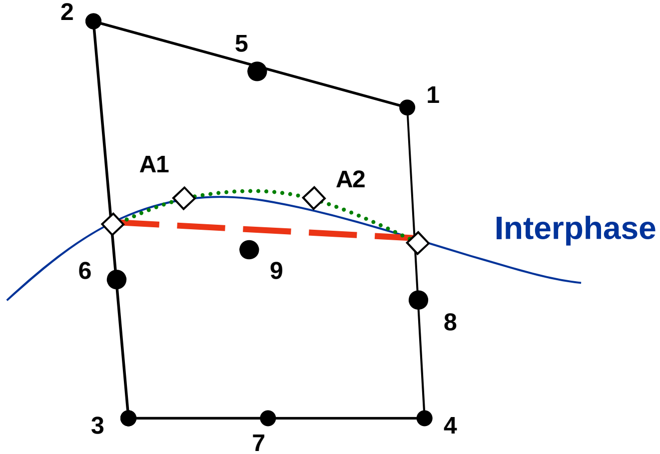
d represents the distance to the interface

σ is an arbitrary constant.

- Within each element intersected by the interface, the interface is approximated by a plane.

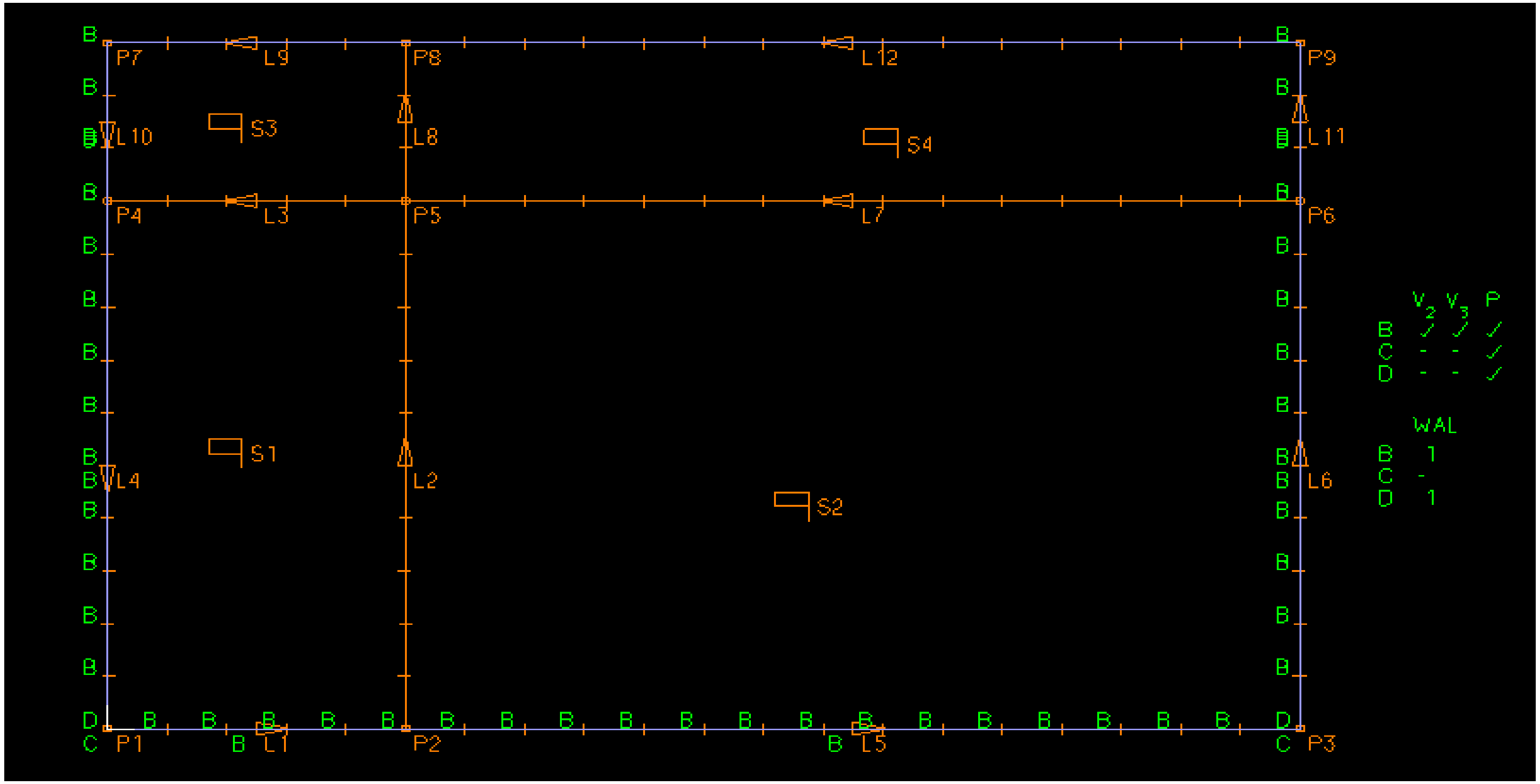
- The distance from any given node in the mesh to each interface plane is calculated.

- The minimum of these distances is taken as *d* for the given node.



Dam breakage

Geometry



Dam breakage

Transient solution