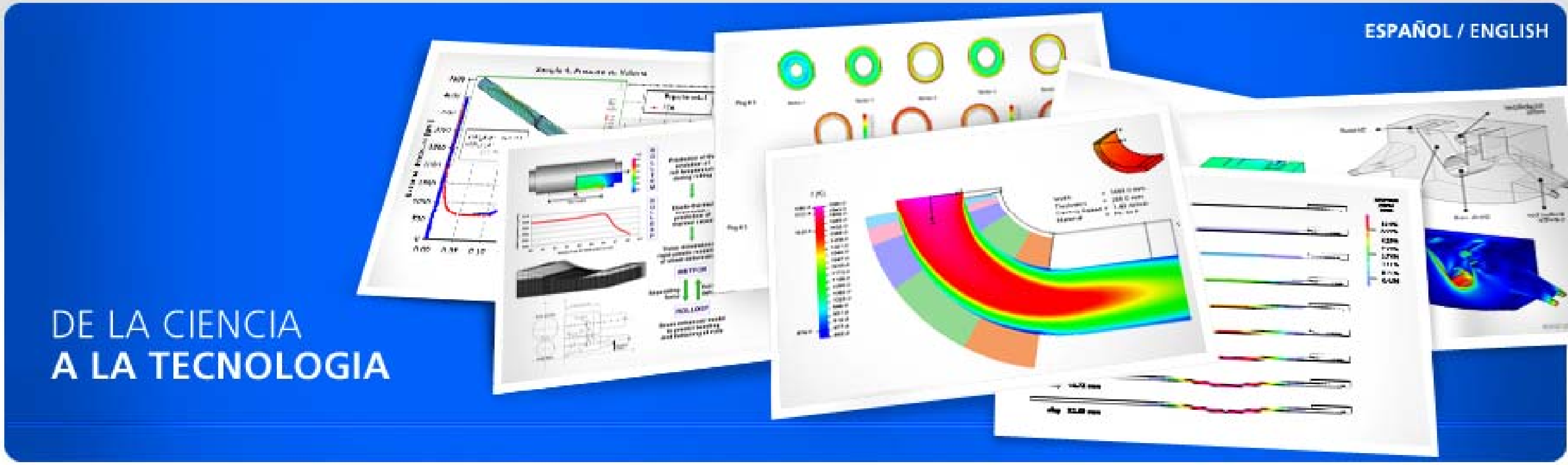
SIM&TEC

Simulación y Tecnología

Simulation and Technology

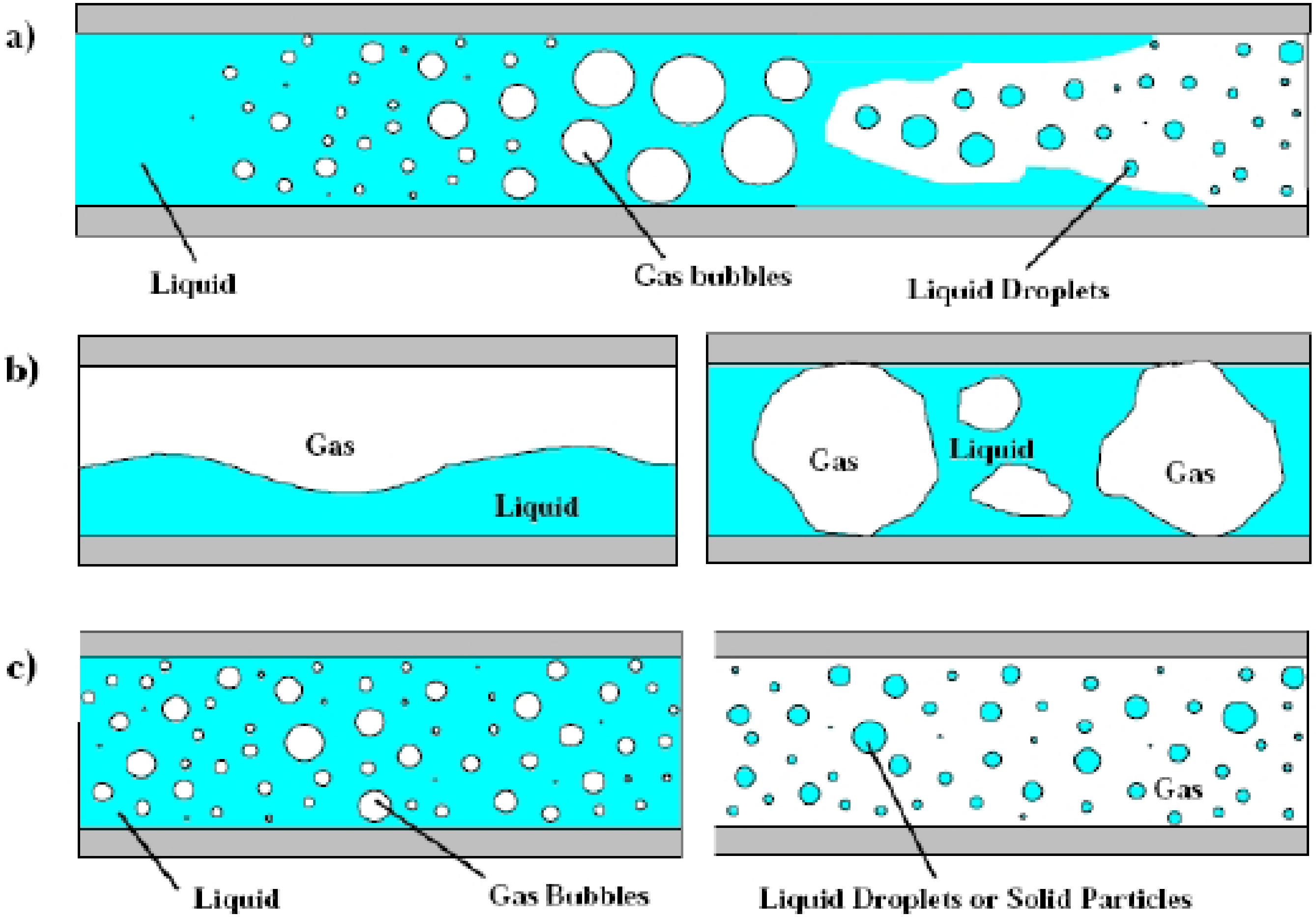


FINITE ELEMENT METHOD IN FLUID DYNAMICS

Part 6: Particles transport model

Marcela B. Goldschmit

Particles transport model



Particles transport model

Lagrangean Model

The particles movement equations are solved. The trajectory of each particles can be obtained

Eulerean Model

The particles concentration equation are solved at each time and each point of the domain.

Particles transport model

- Volumetric fraction of the disperse phase

$$\alpha_p = \frac{\sum_i N_i V_{pi}}{V}$$

- Volumetric fraction of the continuous phase

$$\alpha_f = (1 - \alpha_p)$$

- "Bulk density" of the disperse phase

$$\rho_p^b = \alpha_p \rho_p$$

Particles transport model

- "Bulk density" of the continuous phase

$$\rho_f^b = (1 - \alpha_p)\rho_f$$

- Mix density

$$\rho_m = \rho_f^b + \rho_p^b$$

- Particle concentration

$$n_p = \frac{N_p}{V}$$

Particles transport model

- "Mass loading" (relationship between the mass flow)

$$\eta = \frac{\alpha_p \, \rho_p \, u_p}{(1 - \alpha_p) \, \rho_f \, u_f}$$

- Distance between the particle centers (cubic)

$$\frac{L}{D_p} = \left(\frac{\pi}{6 \, \alpha_p} \right)^{\frac{1}{3}}$$

Particles transport model

Flow classification

$$\alpha_p = \frac{\sum_i N_i V_{Pi}}{V}$$

One-Way coupling: in this regime the influence of the particle phase on the fluid flow may be neglected.

$$\alpha_P < 10^{-6}$$

Two-Way coupling: in this regime the influence of the particle phase on the fluid flow needs to be considered.

$$10^{-6} < \alpha_P < 10^{-3}$$

four-Way coupling: in this regime additional interparticle interactions such as collisions and fluid dynamic interactions between particles become important.

$$\alpha_P > 10^{-3}$$

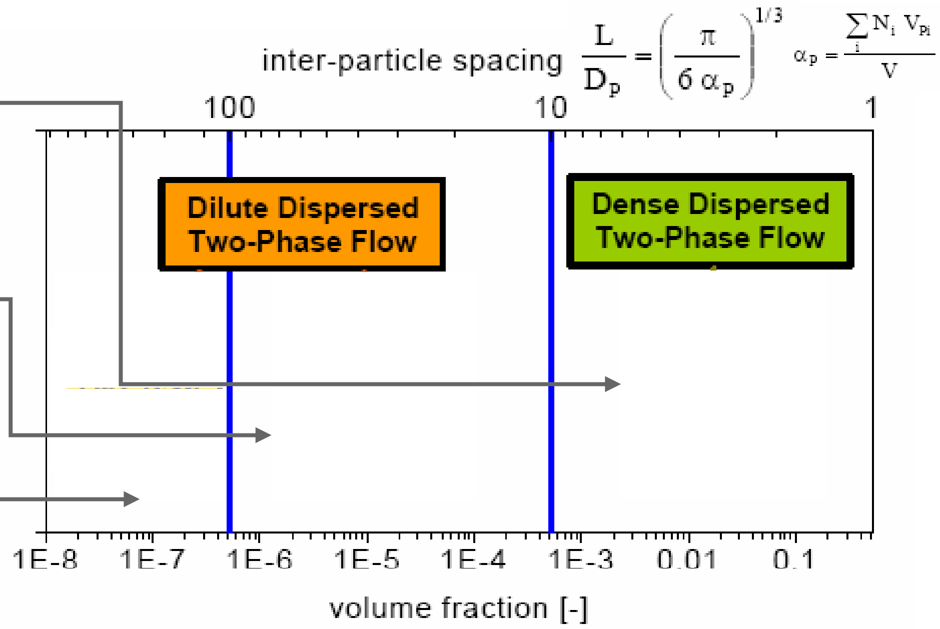
Particles transport model

Flow classification

Fully coupled flow: in this regime particles have influence on the flow and inteparticle interactions are of importance.

Two-Way coupled flow: in this regime the influence of the particle phase on the fluid flow needs to be accounted for.

One-Way coupled flow: in this regime the influence of the particle phase on the fluid flow may be neglected.



<u>One way coupling will be considered for both models</u>

Particles transport model

BBO equation

Basset-Boussinesq-Oseen equation: It is a Newton equation for particle velocity. Different volumetric forces are considered or modeled

$$\rho_p \frac{d\underline{v}}{dt} = \rho \frac{d\underline{u}}{dt} + (\rho_p - \rho)\,\underline{g} - \frac{3}{4}\frac{\rho}{D} c_D (\underline{u}-\underline{v})\left|\underline{u} - \underline{v}\right| - \frac{\rho}{2}\frac{d(\underline{v}-\underline{u})}{dt} + F_{Basset}$$

Inertia force:

It takes into account the acceleration of the carrier phase. Only relevant if the fluid acceleration is strong.

Drag force:

In most fluid-particle system the drag force is dominating the particle motion. It represents the viscous friction exerted by the fluid on the particle.

Basset force:

It is an additional force term that represents the effect of the previous history of the particle on its actual dynamics. It is usually neglected.

Bouyancy force:

It is the volumetric force associated to the particle weight and the correcponding lift due to pressure action on the particle

Added mass force:

It is an additional force which considers the fact that the fluid surrounding the particle is also accelerated.

ρ = *fluid density* ρ_p = *particle density* u = *fluid velocity* v = *particle velocity*

D = *particle diameter* g = *gravity acceleration* C_D = *Drag coefficient.*

Particles transport model

Drag force

$$F_{Drag} = \frac{3}{4}\frac{\rho}{D} c_D (\underline{u} - \underline{v}) |\underline{u} - \underline{v}| \quad \text{or} \quad F_{Drag} = \frac{3}{4} c_D \operatorname{Re}_p \frac{\mu}{D^2} (\underline{u} - \underline{v}) \qquad \operatorname{Re}_p = \frac{D\rho}{\mu} |\underline{u} - \underline{v}|$$

C_D is the drag coefficient which is defined in terms of the particle Reynolds number Re_p

- *for $Re_P < 0.5$*

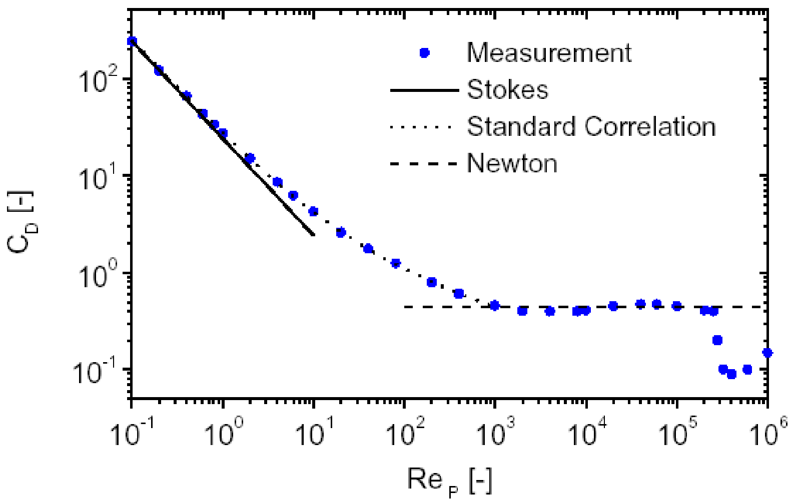
$$C_D = \frac{24}{\operatorname{Re}_p} \qquad F_{Drag} = 18 \frac{\mu}{D^2} (\underline{u} - \underline{v})$$

- *for $0.5 < Re_P < 1000$*

$$c_D = \frac{24}{\operatorname{Re}_P} \left(1 + 0.15 \operatorname{Re}_P^{0.687}\right) = \frac{24}{\operatorname{Re}_P} f_D$$

- *for $1000 < Re_P < 250.000$*

$$C_d \approx 0.44$$



Particles transport model

Drag force

Lagrangean formulation

BBO equation is solved for each particle.

Particle trajectories are determined as consequence of BBO equation solution.

Drag, Buoyancy and Added mass forces are considered.

Particle-wall collision are taken into account

Turbulence effect is introduced by means of a random walk model.

Eulerean formulation

BBO equation is solved to determine particle terminal velocity

Particle distribution is calculated by solving a transport equation (particle trajectories are not calculated)

***Inertial**, Drag, Buoyancy and Added mass forces are considered.*

*Particle-wall collisions are **not** considered in the model*

Turbulence effect is introduced by a diffusive in the transport equation

Particies transport model

Lagrangean particles transport

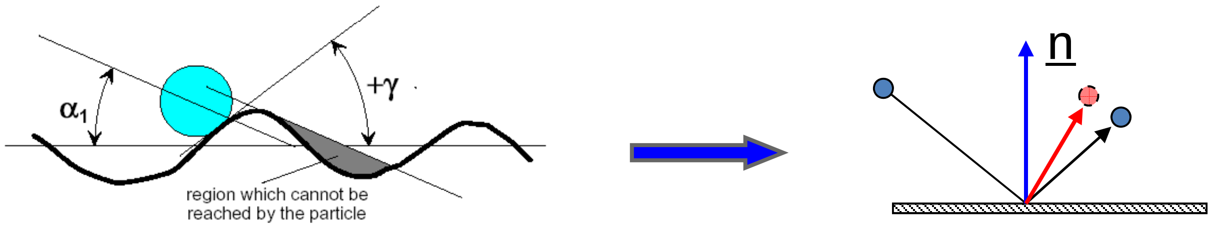
Turbulence is included with a discrete random walk model

$$\underline{u}'_p = \underline{\xi}\sqrt{\frac{2k}{3}}$$ Random particle velocity

$\xi_i \longrightarrow$ Random number normally distributed between -1 and 1

Particle wall collisions

The model allows to consider wall rugosity by means of a random perturbation of the specular reflection



Particles transport model

Lagrangean particles transport

Particle velocity is calculated from the contributions of a term resulting from the solution of the BBO equation and a second term that takes into account turbulent fluctuations of the flow field.

$$\underline{v} = \underline{v}_{BBO} + \underline{v}_{turb}$$

BBO equation is solved for each particle using a backward Euler scheme

Particle velocity due to turbulent fluctuations is obtained with a discret random walk model. Two particles with same initial conditions may have different trajectories. When many particles are considered, an effective diffusion results from turbulence effects.

Taking into account both the mean velocity and the turbulent velocity, particle position is updated accoridng to

$$\underline{x}^{t+\Delta t} = \underline{x}^{t} + \underline{v}_{BBO}^{t+\Delta t/2}\ \Delta t + \sqrt{\Delta t\, \nu_t}\left(\underline{w}^{n+1} - \underline{w}^{n}\right)$$

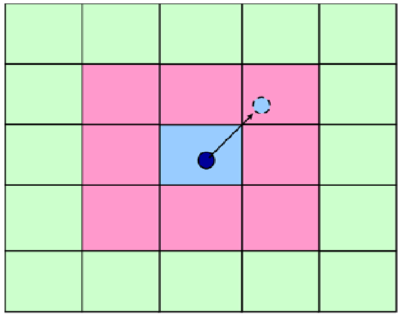
$$\nu_T = C_\mu k^2 \varepsilon^{-1} \sigma_d^{-1}$$

where *w* is random variable with normal gaussian distribution

Particles transport model

Lagrangean particles transport – particles localization

To know the fluid speed in the new particle position it is necessary to know in which element the particle is

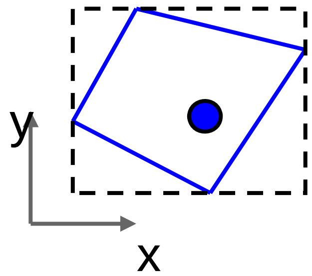


-It is looked initially among the closest neighbors

-First, compare the particle position with the max and min element coordinates (black dashed rectangle)

-Then, obtain the element natural coordinates (r,s) for the particle position solving a nonlinear system

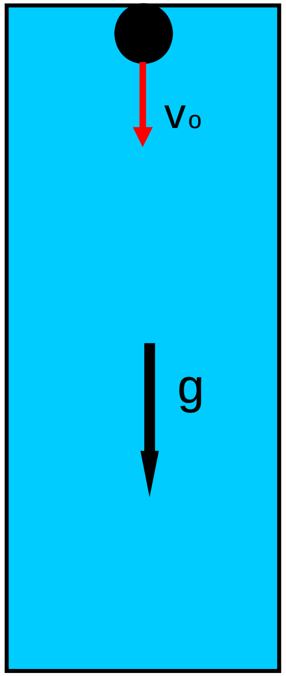
-If -1<(r,s)<1 the particle is in the interior of the element, if not other close elements are analyzed.



Particles transport model

Lagrangean particles transport – model verification

Iron sphere falls into water



Analytical solution

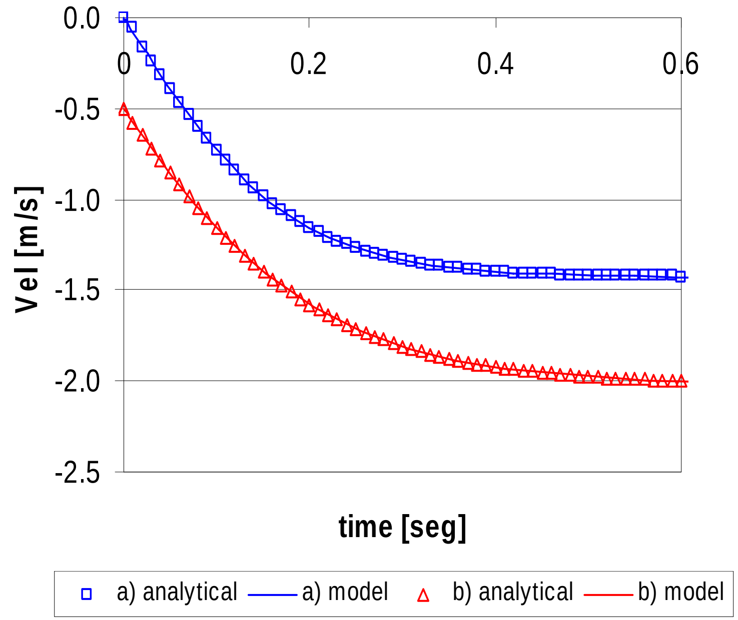
$$v(t) = k\frac{1+c\,e^{-pt}}{1-c\,e^{-pt}}$$

$$c = \frac{v_o - k}{v_o + k}$$

$$b = \frac{C_D\,A_p\,\rho}{2}$$

$$k^2 = \frac{V_p(\rho_p - \rho)}{b}g$$

$$p = 2k\frac{b}{V_p(\rho_p + c_A\,\rho)}$$

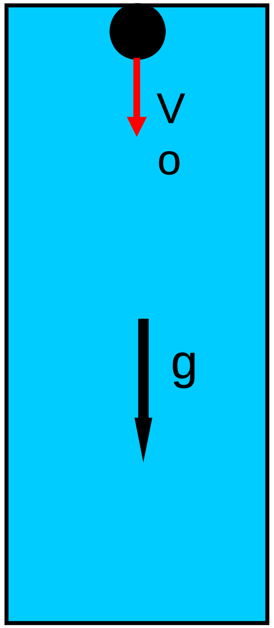


a) $\begin{cases} Dp = 0.01m \\ \rho_p = 7870Kg/m^3 \\ Vo = 0.0m/s \end{cases}$ b) $\begin{cases} Dp = 0.02m \\ \rho_p = 7870Kg/m^3 \\ Vo = 0.5m/s \end{cases}$

Particles transport model

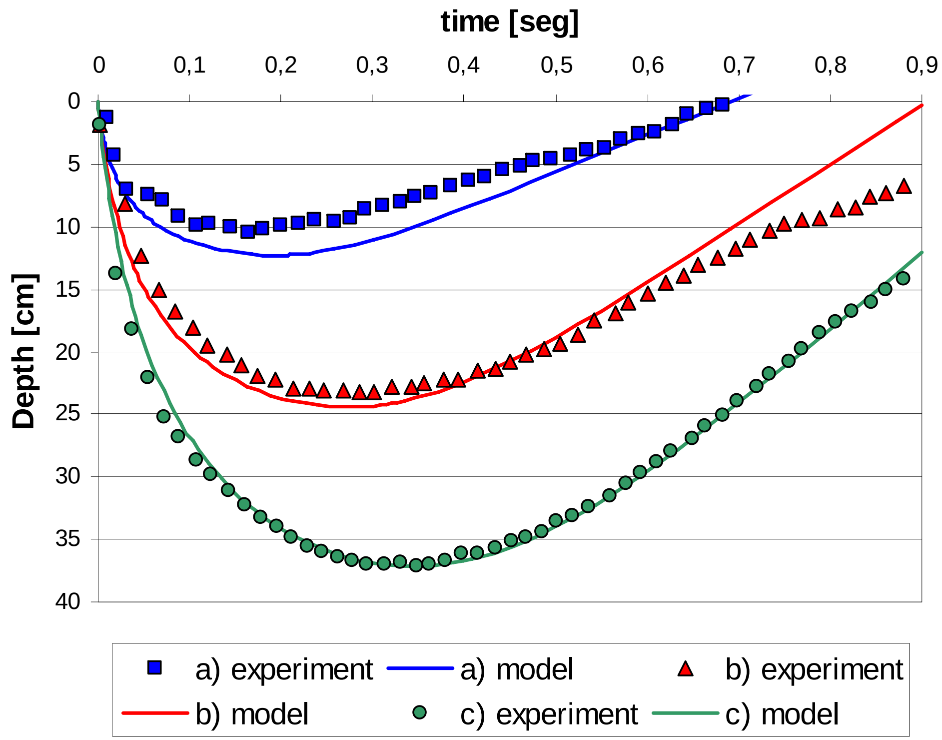
Lagrangean particles transport – model verification

Wooden sphere falls into water



$$\rho_p < \rho$$

$$V_0 \neq 0$$



a) $\begin{cases} Dp = 0.0107m \\ \rho_p = 716Kg/m^3 \\ Vo = 7.74m/s \end{cases}$ b) $\begin{cases} Dp = 0.0269m \\ \rho_p = 711Kg/m^3 \\ Vo = 8.09m/s \end{cases}$ c) $\begin{cases} Dp = 0.0488m \\ \rho_p = 727Kg/m^3 \\ Vo = 8.21m/s \end{cases}$

Particles transport model

Eulerean particles transport - Equations

Mass conservation for particles $$\frac{\partial C}{\partial t} + \nabla \cdot (C\underline{v}) = 0$$ *where C is the mass fraction*

Finite element formulation

- Isoparametric finite elements.
- Streamline Upwind Petrov Galerkin.
- Trapezoidal rule for time discretization

BBO equation used for particle terminal velocity

$$\rho_p \frac{d\underline{v}}{dt} = \rho \frac{d\underline{u}}{dt} + (\rho_p - \rho)\,\underline{g} - \frac{18\mu}{D^2}(\underline{v} - \underline{u}) - \frac{\rho}{2}\frac{d(\underline{v} - \underline{u})}{dt}.$$

Particles transport model

Eulerean particles transport - Equations

- Terminal velocity including turbulent effects

$$\underline{v}_t = \underline{u} + \tau_p \left(\underline{g} - \frac{d\underline{u}}{dt} \right) - \frac{\mu^t}{\rho} \frac{\nabla C}{C} \qquad \tau_p = D^2(\rho_p - \rho)/18\mu$$

- Mass conservation eq. using expression for terminal velocity.

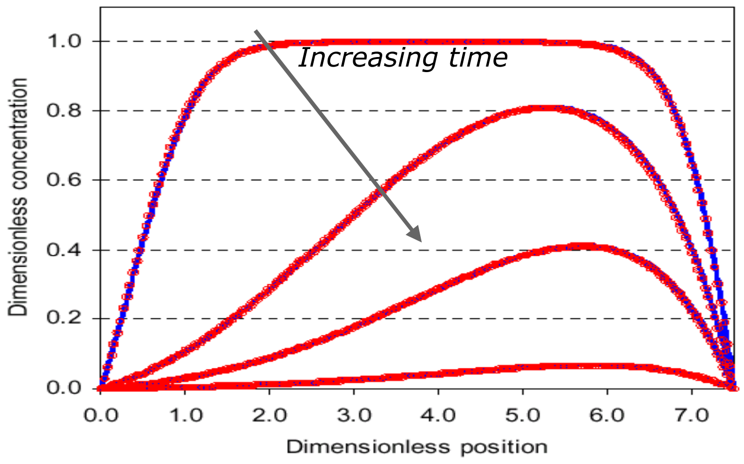
$$\frac{\partial C}{\partial t} + \tau_p C \, \nabla \cdot \left(\frac{d\underline{u}}{dt} \right) + \left(\underline{u} + \tau_p \underline{g} + \tau_p \frac{d\underline{u}}{dt} \right) \cdot \nabla C = \nabla \cdot \left(\frac{C_\mu}{\sigma_d} \frac{k^2}{\varepsilon} \nabla C \right)$$

Particles transport model

Eulerean particles transport - Validation

Three cases where considered for a cylinder of height h and radius R. In all cases uniform initial concentration C_0 and uniform turbulent viscosity m_T were assumed.

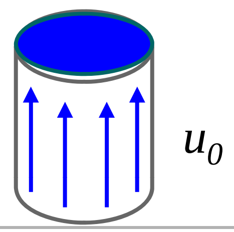
- Example 1: Uniform velocity in axial direction (z) u0 with C=0 on z=0 and z=h



$$C = C_0 \sum_{m=1}^{\infty} \frac{1 - e^{-\frac{1}{2}L}(-1)^m}{1 + 4k_m^2} \frac{8k_m}{L} \exp\left(-\left(k_m^2 + \frac{1}{4}\right)\theta\right) \exp\left(\frac{\zeta}{2}\right) \sin\left(k_m \zeta\right)$$

$$\theta = \frac{t}{\nu_T}\left(u_0 + \tau g\right)^2 \text{ and } \zeta = \frac{z}{\nu_0}\left(u_0 - \tau g\right)$$

$$L = \frac{h}{\nu_T}\left(u_0 - \tau g\right) \qquad k_m = \pi m / L$$

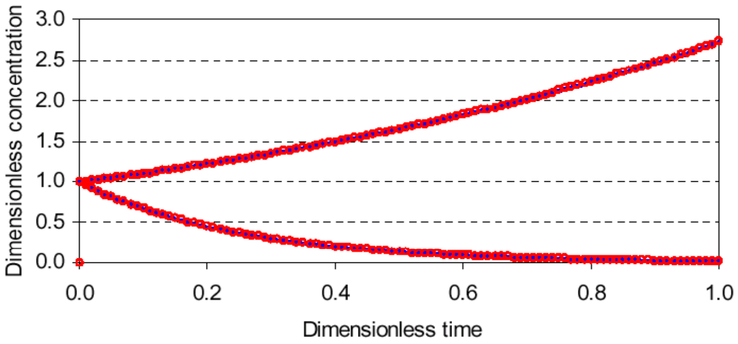


Particles transport model

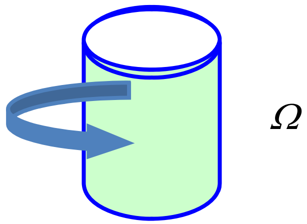
Eulerean particles transport - Validation

Three cases where considered for a cylinder of height h and radius R. In all cases uniform initial concentration C_0 and uniform turbulent viscosity m_T were assumed.

- *Example 2*: Uniform rotation with angular velocity *W.* with no flux on r=R.



$$C_N = C_0 \exp\left(-2\tilde{\tau}\Omega^2 t\right)$$

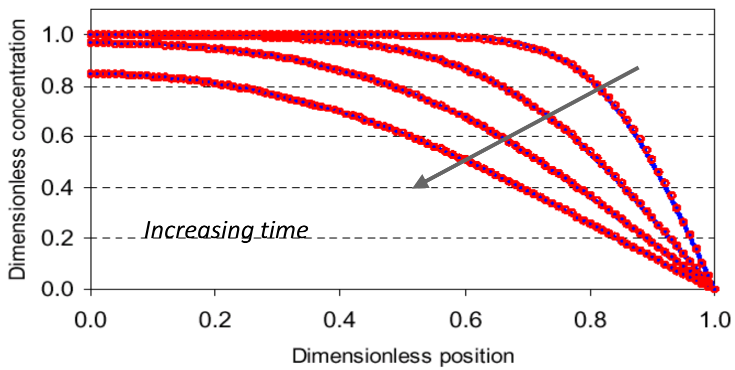


Particles transport model

Eulerean particles transport - Validation

Three cases where considered for a cylinder of height h and radius R. In all cases uniform initial concentration C_0 and uniform turbulent viscosity m_T were assumed.

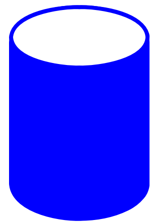
- *Example 3*: Fluid at rest. Pure diffusion with *C=0* on *r=R*.



$$C_D = \sum_n C_n \exp\left(-\nu_0 \frac{z_n^2}{R^2}\, t\right) J_0\left(r\frac{z_n}{R}\right)$$

$$C_n = 2C_0 \left(z_n J_1(z_n)\right)^{-2} \int_0^{z_n} J_0(\xi)\xi d\xi$$

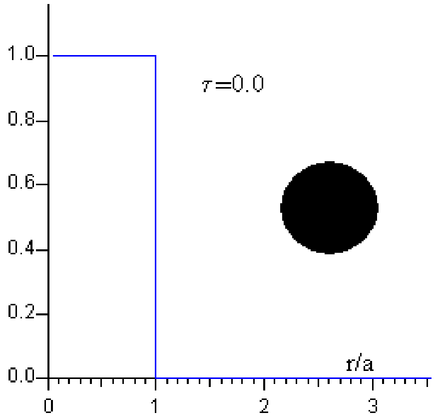
z_n are the zeros of the Bessel functions J_0.



Particles transport model

Axisymmetric homogenous diffusion in quiet liquid.

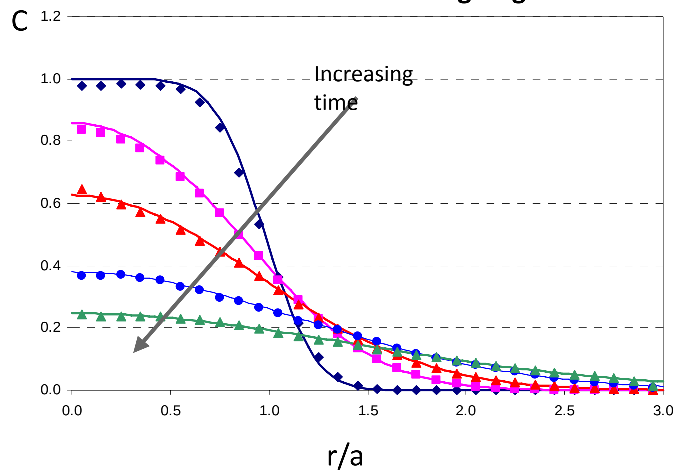
Initial condition.



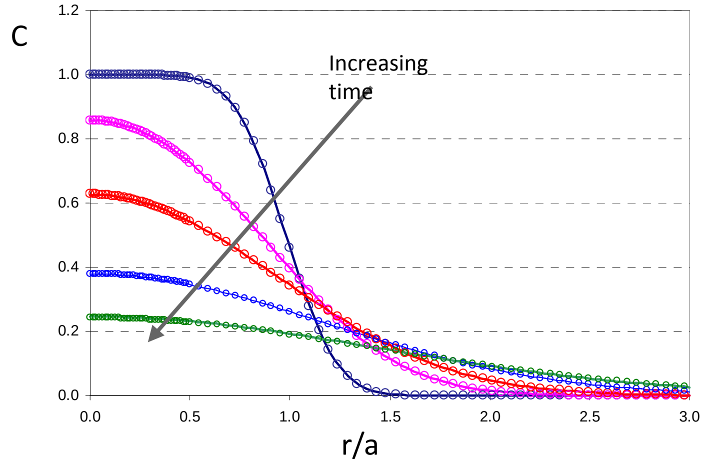
Analytical solution.

$$C(r,t) = \frac{1}{2Dt}\exp\left(\frac{-r^2}{4Dt}\right)\int_0^a \exp\left(\frac{-z^2}{4Dt}\right) I_0\left(\frac{r\,z}{2Dt}\right) z\,dz$$

Lagrangian model



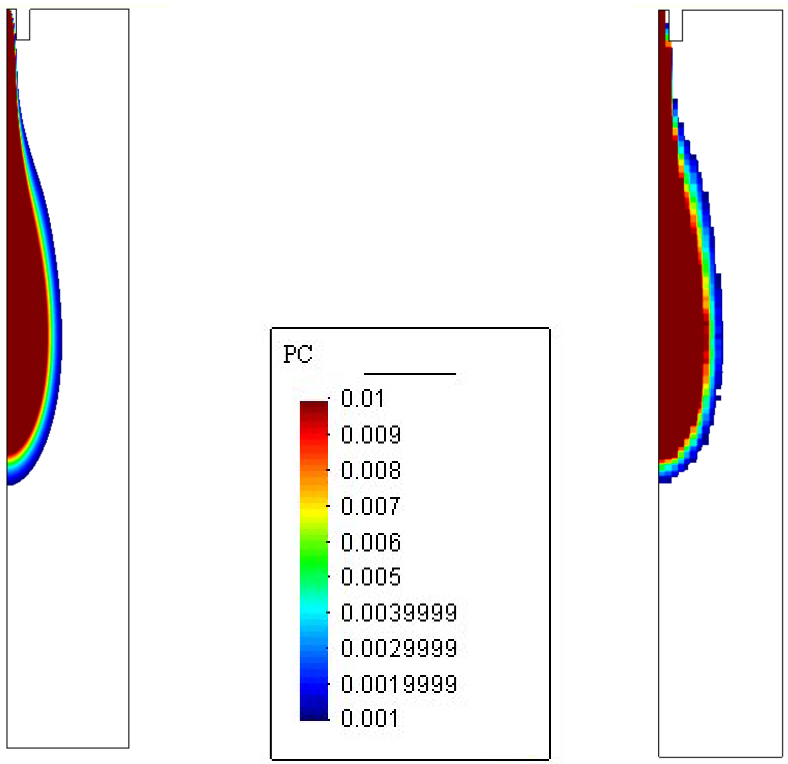
Eulerian model



Particles transport model

Pulse of paticles injected in a mould

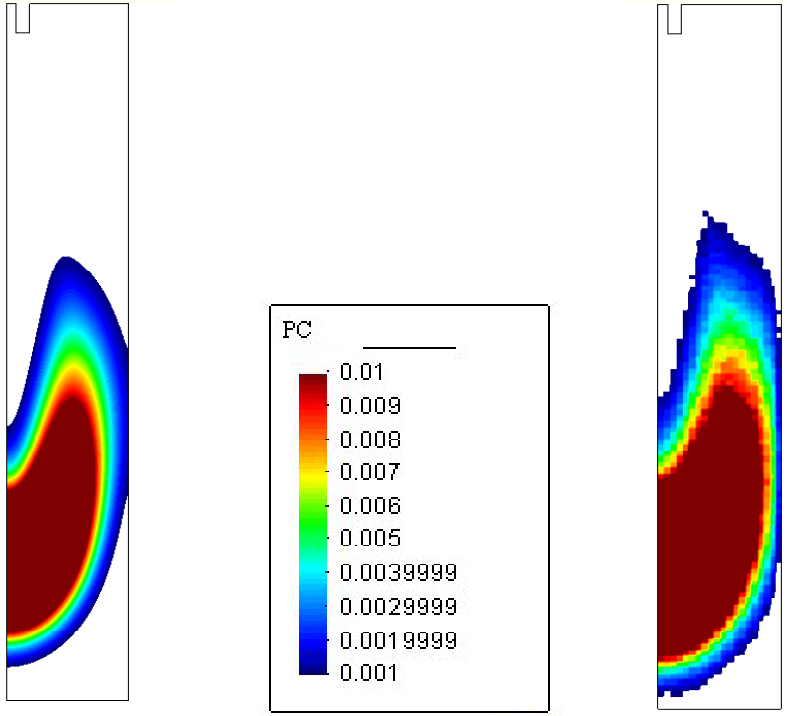
Eulerian simulation



Lagrangian simulation

Particles transport model

Pulse of paticles injected in a mould



Eulerian simulation

Lagrangian simulation

Particles transport model

Dross movement in cincalume pot

The cincalum process generates a covering of Zn-Al on a steel strip that circulates through a melted alloy bath. The cincalum bath generates unwanted particles named “dross particles” that produce defects in the coating. The lagrangean particle transport numerical modelling is a a powerful and reliable tool for simulating the movement and deposition of this particles in order to analyze the influence of the different operatives variables.