Effects of internal/external pressure on the global buckling of pipelines

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Abstract

The global buckling (Euler buckling) of slender cylindrical pipes under internal/external pressure and axial compression is analyzed. For perfectly straight elastic pipes an approximate analytical expression for the bifurcation load is developed. For constructing the nonlinear paths of imperfect (non straight) elasto-plastic pipes a finite element model is developed. It is demonstrated that the limit loads evaluated via the nonlinear paths tend to the approximate analytical bifurcation loads when these limit loads are inside the elastic range and the imperfections size tends to zero.

Keywords: Internal pressure; External pressure; Axial compression; Euler buckling; Pipeline

1. Introduction

When a straight pipe under axial compression and internal (external) pressure is slightly perturbed from its straight configuration there is a resultant force, coming from the net internal (external) pressure, that tends to enlarge (diminish) the curvature of the pipe axis. Hence, for a straight pipe under axial compression, if the internal pressure is higher than the external one, there is a destabilizing effect due to the resultant pressure load and therefore, the pipe Euler buckling load is lower than the Euler buckling load for the same pipe but under equilibrated internal/external pressures; on the other hand when the external pressure is higher than the internal one the resultant pressure load has a stabilizing effect and therefore the pipe Euler buckling load is higher than the Euler buckling load for the same pipe but under equilibrated internal/external pressures.

The analysis of the buckling load of slender cylindrical pipes under the above described loading is important in many technological applications; for example, the design of pipelines. In Fig. 1 we present a simple case, for which the axial compressive load (T) has a constant part (C) and a part proportional to the internal pressure (p_i).

That is to say,

$$T = C + kp_i \tag{1}$$

where k is a constant depending on the particular application.

In the second section of this paper we develop an approximate analytical expression for calculating the Euler buckling load for elastic perfectly straight cylindrical pipes (bifurcation limit load) and in the third section we develop a finite element model to determine the equilibrium paths of imperfect (non straight) elasto-plastic cylindrical pipes. From the analysis of the nonlinear equilibrium paths it is possible to determine the limit loads of pipes under axial compression and internal/external pressure. Of course, this limit loads depend on the pipe imperfections; however, we show via numerical examples that, for the cases in which the bifurcation limit loads are inside the elastic range, the pipe limit loads tend to the bifurcation limit loads when the imperfections size tends to zero.

2. Elastic buckling of perfect cylindrical pipes

2.1. Internal pressure

In Fig. 1 we represent a perfectly straight slender cylindrical pipe, in equilibrium under an axial compressive load and internal pressure; let us assume that we perturb the straight equilibrium configuration getting an infinitely close

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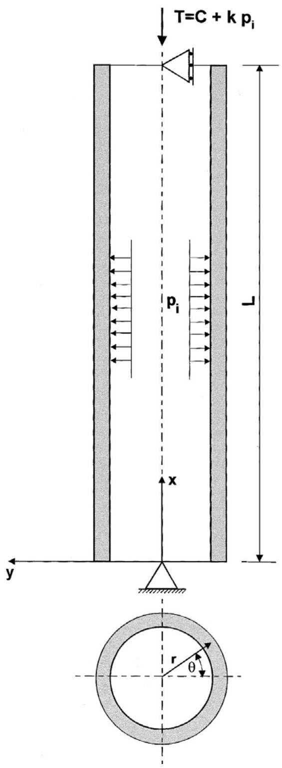

Fig. 1. Cylindrical pipe under internal pressure and axial compression.

configuration defined by the transversal displacement, $v(x)$, of the points on the pipe axis. If for some loading level, defined by p_i and by Eq. (1), this perturbed configuration is in equilibrium we say that the load level is critical (buckling load) because a bifurcation of the equilibrium path, in the loads–displacements space, is possible.

Due to the polar symmetry of the problem we consider that all the displacements $v(x)$ are parallel to a plane. For a longitudinal fiber defined by the polar coordinates (x, r, θ) (see Fig. 1) we have, for the case of small strains,

$$\varepsilon_{xx} = -v''(x) r \cos\theta \tag{2}$$

where ε_{xx} is the axial strain and $v''(x) = \frac{d^2 v(x)}{dx^2}$.

On a differential pipe length, the resultant pressure force due to the pipe bending is normal to the bent axis direction (follower load) and its value is,

$$q(x)\,dx = 2\int_0^{\pi} p_i \cos\theta (1 + \varepsilon_{xx}) r_i \,d\theta\,dx \tag{3}$$

where r_i is the pipe inner radius.Using Eqs. (2) and (3) we get,

$$q(x) = -p_i \pi r_i^2 v''(x) \tag{4}$$

which is the resulting force per unit length produced by the internal pressure acting on the deformed configuration. This load per unit length has horizontal and vertical components that in our case ($v'(x) \ll 1$) are,

$$q_h(x) = q(x)\cos\left[v'(x)\right]; \quad q_v(x) = q(x)\sin\left[v'(x)\right]. \tag{5}$$

Using a series expansion of the trigonometric functions and neglecting higher order terms, we get:

$$q_h(x) = -p_i \pi r_i^2 v''(x); \quad q_v(x) = 0. \tag{6}$$

To analyze the equilibrium of the perturbed configuration, being this an elastic problem, we use the Principle of Minimum Potential Energy [1,2]. When only conservative loads are acting on the pipe, equilibrium is fulfilled if, in the perturbed configuration,

$$\delta\Pi = 0 \tag{7}$$

where Π is the potential energy,

$$\Pi = U - V \tag{8}$$

U: elastic energy stored in the pipe material, V: potential of the external conservative loads.

In our case we have to consider the displacement dependent loads (non-conservative) given by Eq. (6), therefore [3]:

$$\delta(U - V) - \int_0^L q_h \delta v(x)\,dx = 0 \tag{9}$$

and [1],

$$U = \frac{EI}{2}\int_0^L \left[v''(x)\right]^2 dx, \tag{10a}$$

$$V = \frac{T}{2}\int_0^L \left[v'(x)\right]^2 dx, \tag{10b}$$

$$\int_0^L q_h\,\delta v(x)\,dx = \int_0^L -p_i \pi r_i^2 v''(x)\,\delta v(x)\,dx \tag{10c}$$

E: Young's modulus of the pipe material, I: inertia of the pipe section with respect to a diametral axis.

Hence, introducing the above in Eq. (9) we get for the fulfillment of equilibrium,

$$\delta\left[\frac{EI}{2}\int_0^L \left[v''(x)\right]^2 dx - \frac{T}{2}\int_0^L \left[v'(x)\right]^2 dx\right] + p_i \pi r_i^2 \int_0^L v''(x)\delta v(x)\,dx = 0. \tag{11}$$

We search for an approximate solution of the above equation using the Ritz Method [1], therefore we try as an approximate solution,

$$\tilde{v}(x) = \sum_{n=1,2,\ldots} a_n \sin\frac{n\pi x}{L}. \tag{12}$$

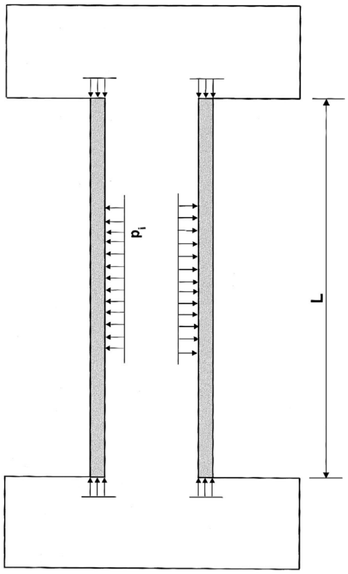

Fig. 2. Simply supported pipe open on both ends under internal pressure.

Introducing the proposed approximate solution in Eq. (11) and taking into account that the a_n are arbitrary constants we get for equilibrium,

$$\left[\frac{EIn^4\pi^4}{L^3} - \frac{Tn^2\pi^2}{L} - p_i r_i^2 \frac{n^2\pi^3}{L}\right] a_n = 0 \qquad n = 1, 2, \ldots \tag{13}$$

The above equations have two possible solution sets:

- $a_n = 0$; which corresponds to the unperturbed straight configuration.
- $\left[\frac{EIn^4\pi^4}{L^3} - \frac{Tn^2\pi^2}{L} - p_i r_i^2 \frac{n^2\pi^3}{L}\right] = 0$; which corresponds to an equilibrium configuration different from the straight one.

The second solution gives the location of the bifurcation point (critical loading),

$$T_{cr} + p_{icr}\pi r_i^2 = \frac{n^2 EI\pi^2}{L^2} \tag{14a}$$

$$C_{cr} + kp_{icr} + p_{icr}\pi r_i^2 = \frac{n^2 EI\pi^2}{L^2}. \tag{14b}$$

It is interesting to realize that the above equations predict that there is a critical (buckling) pressure also if there is no axial compression ($T = 0$) and even if there is axial tension on the pipe ($T < 0$).

Let us consider the following cases:

- Simply supported pipe, closed on both ends, under internal pressure.

In this case, $C = 0$ and $k = -\pi r_i^2$; hence, from Eq. (14b) it is obvious that the only possible solution is the straight configuration and no bifurcation is possible.

- Simply supported pipe, open on both ends, under internal pressure (Fig. 2).

An example of this case is the hydraulic testing of a pipe. In this case: $C = 0$, $k = \pi\left(r_e^2 - r_i^2\right)$. Hence, using Eq. (14b) we get,

$$p_{icr} = \frac{EI\pi}{L^2 r_e^2}.$$

Obviously, if there are $(n - 1)$ intermediate supports we have,

$$p_{icr} = \frac{n^2 EI\pi}{L^2 r_e^2}.$$

2.2. External pressure

For the cases in which the pipe is submitted to external pressure we rewrite Eq. (6) as,

$$q_h(x) = p_e \pi r_i^2 v''(x); \qquad q_v(x) = 0. \tag{15}$$

Hence, after some algebra we get for the equilibrium of the perturbed configuration,

$$\delta\left[\frac{EI}{2}\int_0^L \left[v''(x)\right]^2 \mathrm{d}x - \frac{T}{2}\int_0^L \left[v'(x)\right]^2 \mathrm{d}x\right] - p_e\pi r_i^2 \int_0^L v''(x)\,\delta v(x)\,\mathrm{d}x = 0 \tag{16}$$

using as an approximation for the equilibrium configuration the one written in Eq. (12), we finally get,

$$\left[\frac{EIn^4\pi^4}{L^3} - \frac{Tn^2\pi^2}{L} + p_e r_i^2 \frac{n^2\pi^3}{L}\right] a_n = 0 \qquad n = 1, 2, \ldots \tag{17}$$

therefore, for the nontrivial solution,

$$T_{cr} - p_{ecr}\pi r_i^2 = \frac{n^2 EI\pi^2}{L^2} \tag{18a}$$

$$C_{cr} + kp_{ecr} - p_{ecr}\pi r_i^2 = \frac{n^2 EI\pi^2}{L^2}. \tag{18b}$$

From the above equations it is obvious that the external pressure has a stabilizing effect on the pipe; that is to say, the axial compressive load that makes the pipe buckle is higher than the Euler load of the pipe under equilibrated internal/external pressures.

Let us consider the following case:

- Simply supported pipe, closed on both ends, under external pressure.

For this case $C = 0$ and $k = \pi r_e^2$ therefore from Eq. (18b) we get,

$$p_{ecr} = \frac{EI\pi}{L^2(r_e^2 - r_i^2)}$$

and if the pipe has $(n - 1)$ intermediate supports,

$$p_{ecr} = \frac{n^2 EI\pi}{L^2(r_e^2 - r_i^2)}.$$

Comparing this result with the one corresponding to the pipe under internal pressure it is obvious that the pipe under external pressure can withstand a higher pressure without reaching the bifurcation load; hence, it is obvious the stabilizing effect of the external pressure.

3. Nonlinear equilibrium paths for non-straight elasto-plastic cylindrical pipes

An actual pipe is not perfectly straight, and its random imperfections will have a projection on the buckling mode of the perfect pipe; hence, when analyzing the equilibrium path of a non-perfect pipe we shall encounter a limit point rather than a bifurcation point [4]. The load level of this limit point shall depend on the pipe imperfections, will be lower than the bifurcation load of the perfect pipe and will tend to this value when the imperfections size tends to zero.

In order to analyze the nonlinear equilibrium paths of imperfect pipes we developed a finite element model using the general purpose finite element code ADINA [5].

Some basic features of the developed finite element model are:

- The pipe behavior is modelled using Hermitian (Bernoulli) beam elements [6].
- The pipe model is developed using an Updated Lagrangian formulation with an elasto-plastic (associated von Mises) material model (finite displacements and rotations but infinitesimal strains) [6].
- Acting on the beam elements we consider a conservative load (T) and a deformation dependent load normal to the pipe axis, that for the case of internal pressure is (see Eq. (6)),

$$q_h = -p_i \pi r_i^2 [v''(x) + \zeta''(x)]$$

where $\zeta(x)$ is the initial imperfection of the pipe axis.

We simply calculate, in our finite element implementation,the second derivatives using a finite differences scheme. To provide a numerical example, we use the finite element model to analyze the following case:

Pipe outside diameter	60.3 mm
Pipe wall thickness	3.9 mm
Pipe length	12,200 mm
Intermediate grips	4
Pipe yield strength	38.70 kg/mm^2
Hardening modulus	0.0

under the loading defined by an internal pressure and, $C = 0$, $k = \pi\left(r_e^2 - r_i^2\right)$.

3.1. No clearance between the pipe and the grip

We consider the following initial imperfection for the pipe axis,

$$\zeta(x) = \alpha \, 0.2 \frac{L}{1000} \sin\left(\frac{5\pi x}{L}\right) \tag{19}$$

which is obviously zero at the grips and is coincident with the first buckling mode predicted using the Ritz method (Eq. (12)).

In Fig. 3 we plot the load–displacement equilibrium path for various values of α and in the same graph we plot the bifurcation limit load obtained using Eq. (14b).

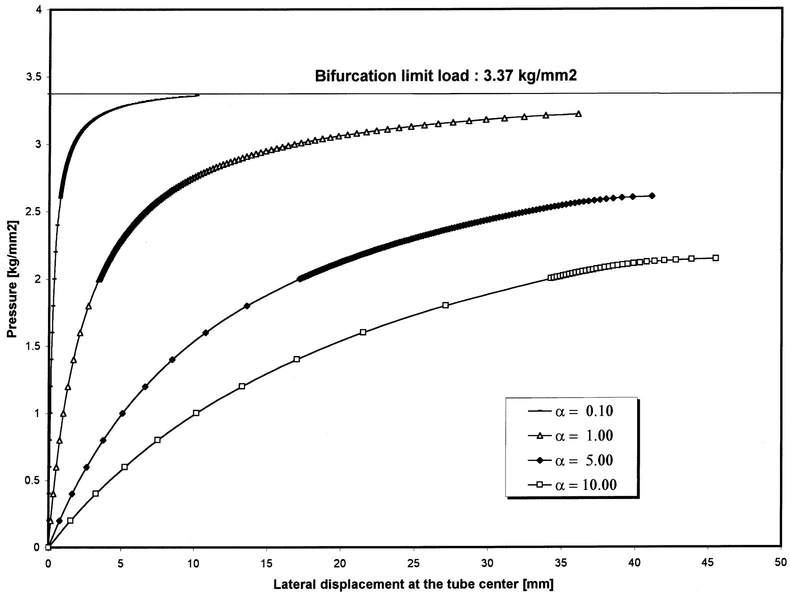

Fig. 3. Grips with no clearance. Load–displacement curves.

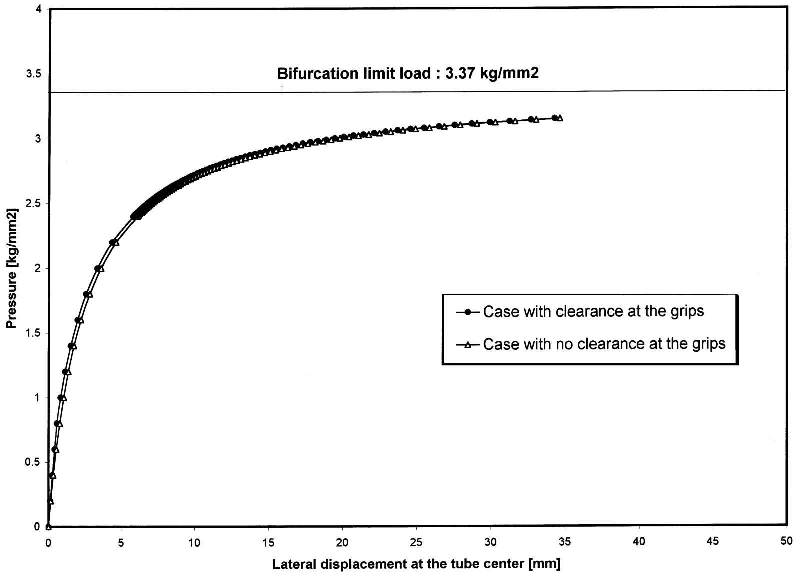

Fig. 4. Clearance between grips and pipe body. Load–displacement curves.

We can verify from this figure that the limit load increases when the size of the imperfection (α) diminishes, and that it tends to the bifurcation limit load when $\alpha \to 0$.

3.2. Clearance between pipe and grips

This is a more realistic case because, unless the grips are welded to the pipe body, there is usually some clearance between the pipe and the grips.

We analyze the same case that was considered in the previous subsection but allowing for a clearance between the grip and the pipe body of 5 mm. We consider the following initial imperfection for the pipe axis,

$$\zeta(x) = 0.2\frac{L}{1000}\sin\left(\frac{5\pi x}{L}\right) + \left(0.2\frac{L}{100} - 0.2\frac{L}{1000}\right)\sin\left(\frac{\pi x}{L}\right) \quad (20)$$

and between the rigid grip and the pipe we introduce a contact condition.

In Fig. 4 we plot the nonlinear equilibrium paths corresponding to the cases:

- Clearance between grips and pipe body (initial imperfection as per Eq. (20)).
- No clearance between grips and pipe body (initial imperfection as per Eq. (19) with $\alpha = 1.0$).

From the results plotted in Fig. 4 it is obvious that the only imperfection that has an influence on the pipe critical load is the imperfection that is coincident with the first pipe buckling mode.

4. Conclusions

We derived an approximate analytical expression for calculating the Euler buckling load of a pipe under axial compression and internal/external pressure. This expression incorporates the destabilizing/stabilizing effect of the internal/external pressure.

We constructed a finite element model to determine the nonlinear equilibrium paths, in the loads–displacements space, of imperfect (non-straight) elasto–plastic pipes. From the analysis of the nonlinear equilibrium paths it is possible to determine the limit loads of pipes under axial compression and internal/external pressure. Of course, these limit loads depend on the pipe imperfections; however, we showed via numerical examples that, for the cases in which the bifurcation limit loads are inside the elastic range, the pipe limit loads tend to the bifurcation limit loads when the imperfections size tends to zero.

Acknowledgements

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